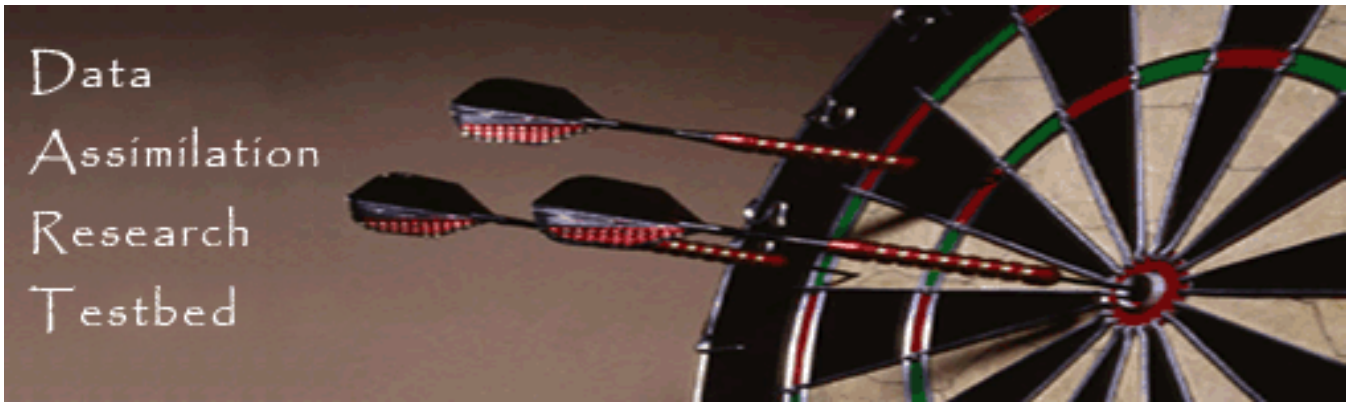
Introduction to Ensemble Kalman Filters and the Data Assimilation Research Testbed



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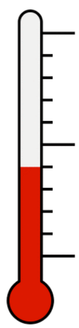
What is Data Assimilation?

Observations combined with a Model forecast…



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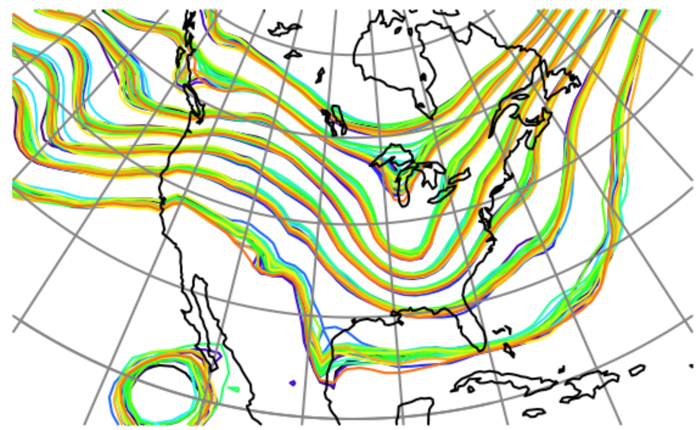
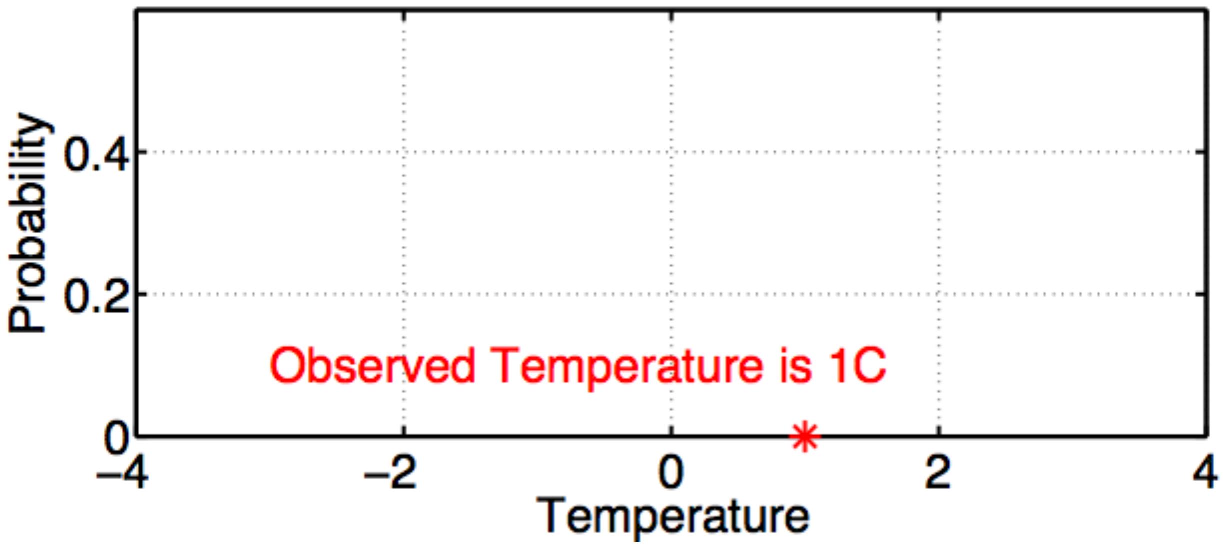


…to produce an analysis
(best possible estimate).

Example: Estimating the Temperature Outside

An observation has a value (*),



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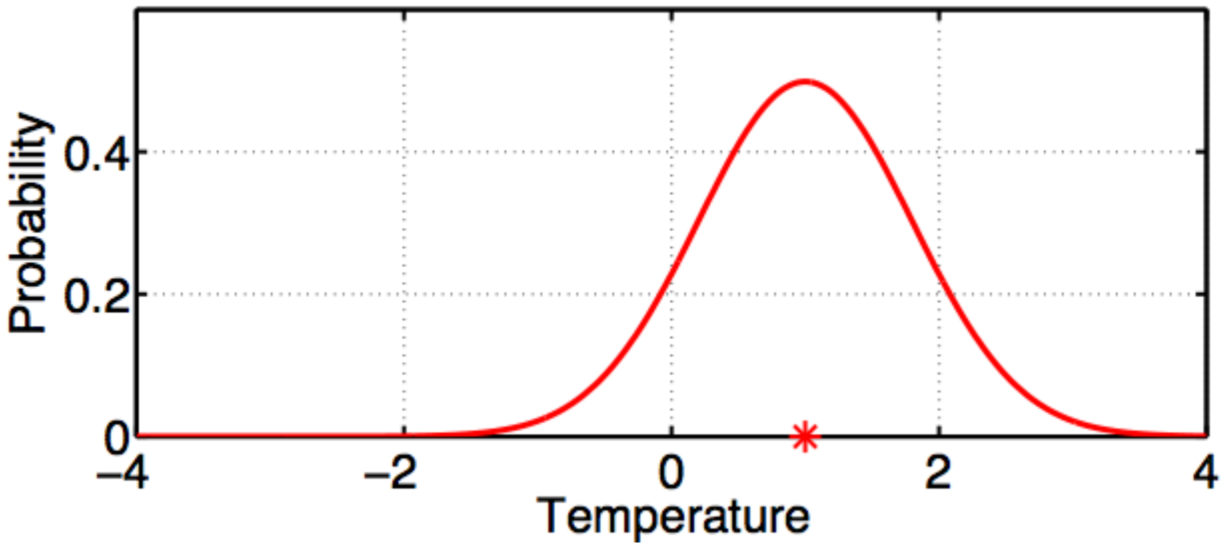
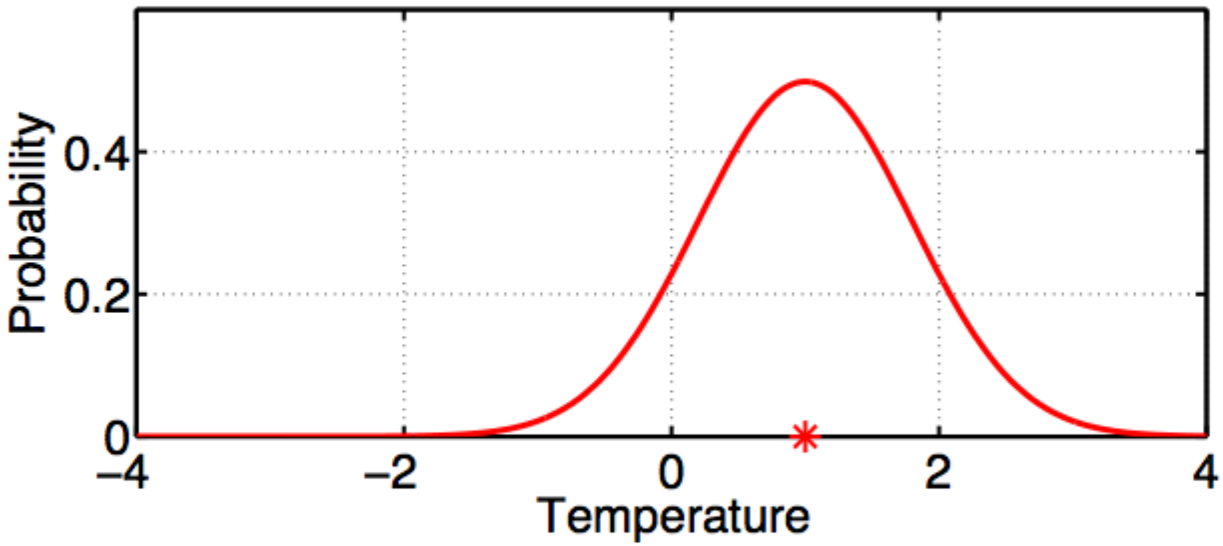
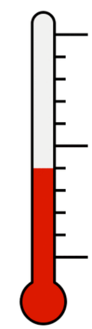


and an error distribution (red curve) that is associated with the instrument.

Example: Estimating the Temperature Outside

Thermometer outside measures 1C.





Instrument builder says thermometer is unbiased with +/- 0.8C gaussian error.

Example: Estimating the Temperature Outside

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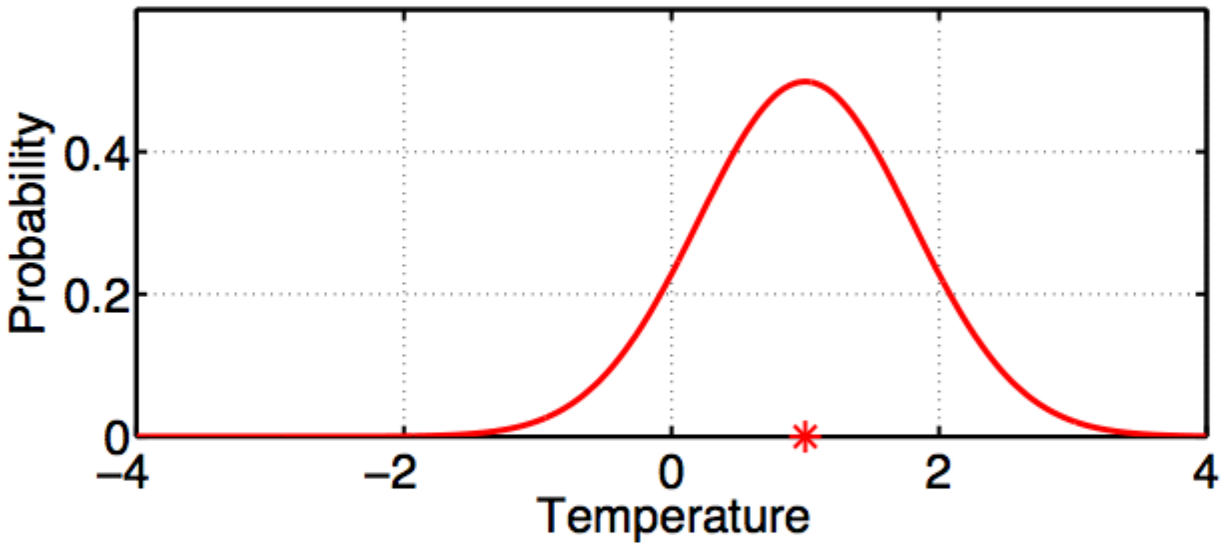
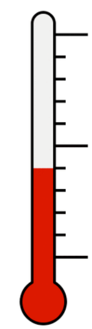
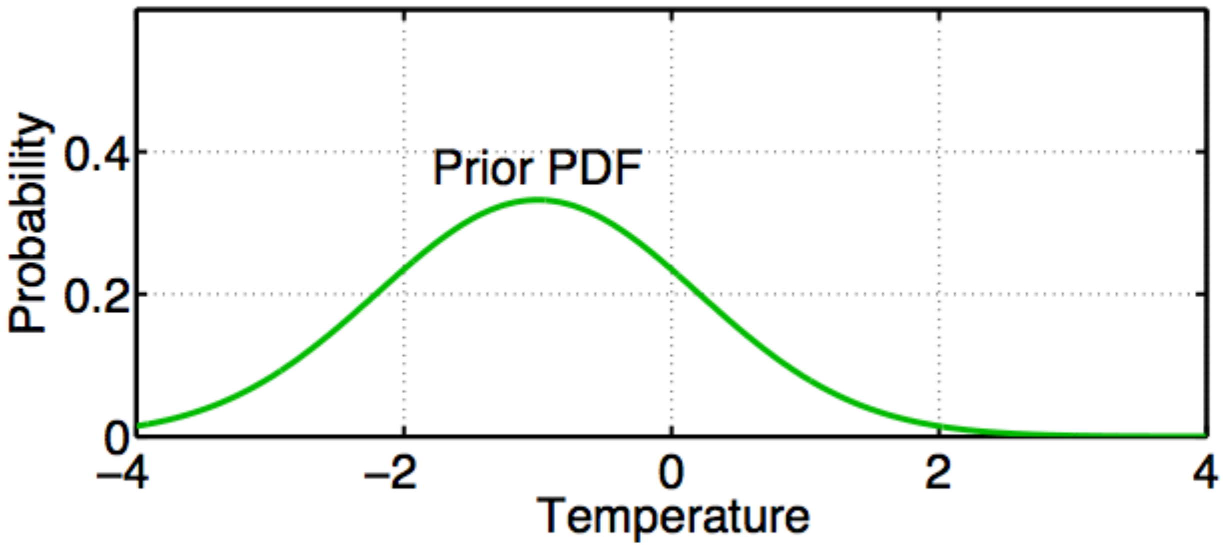




The red plot is $P(T \mid T_o)$, probability of temperature given that T_o was observed.

Example: Estimating the Temperature Outside

We also have a prior estimate of temperature.



The green curve is $P(T \mid C)$; probability of temperature given all available prior information C.

Example: Estimating the Temperature Outside

Prior information C can include:

1. Observations of things besides T;
2. Model forecast made using observations at earlier times;
3. *A priori* physical constraints (T > -273.15C);
4. Climatological constraints (-30C < T < 40C).

Combining the Prior Estimate and Observation

Prior

Bayes Theorem:

$$P(T \mid T_o, C) = \frac{P(T_o \mid T, C) P(T \mid C)}{P(T_o \mid C)}$$

Posterior: Probability of T given observations and Prior. Also called update or analysis.

Likelihood: Probability that T_o is observed if T is true value and given prior information C.

Combining the Prior Estimate and Observation

Rewrite Bayes as:

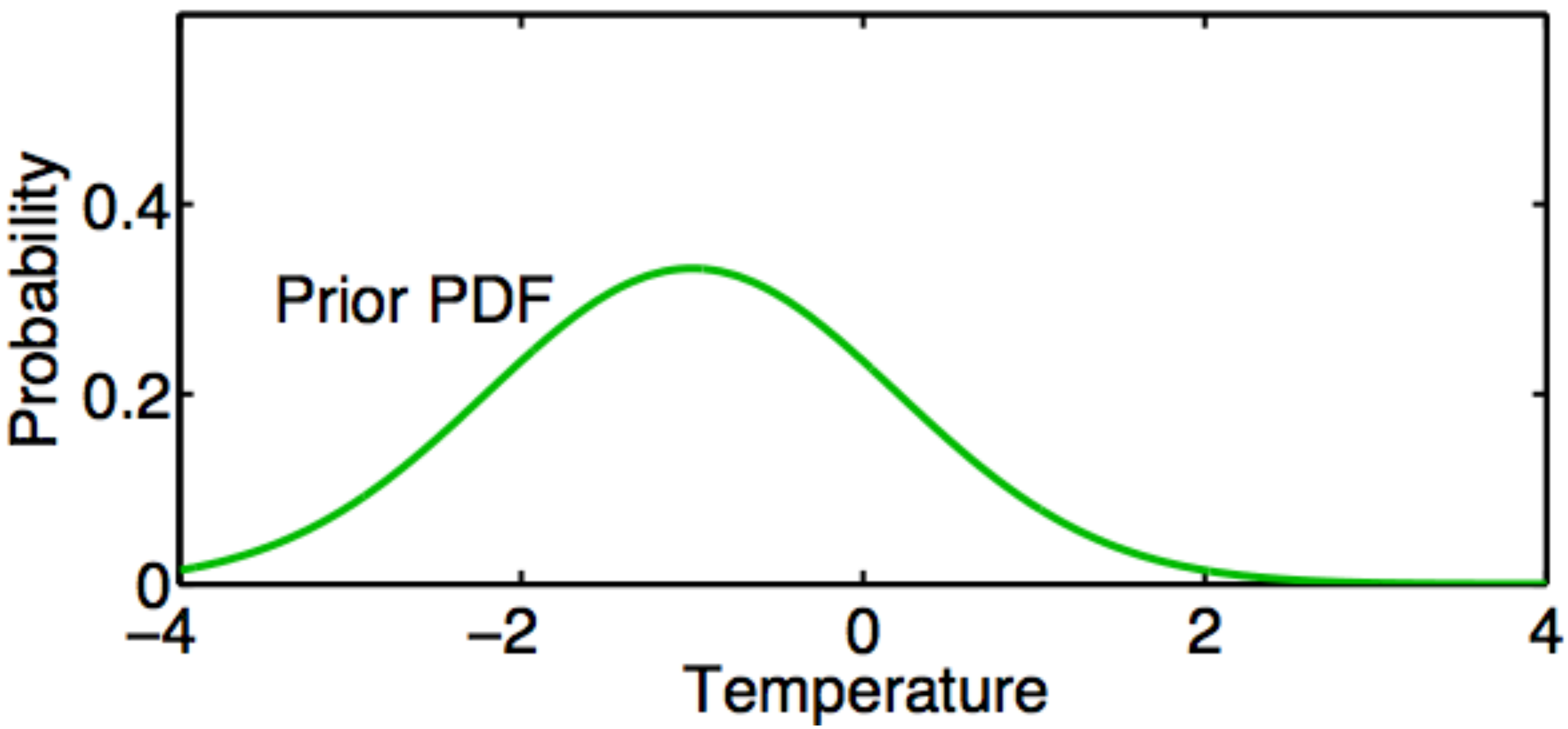
$$\frac{P(T_o \mid T,C)P(T \mid C)}{P(T_o \mid C)} = \frac{P(T_o \mid T,C)P(T \mid C)}{\int P(T_o \mid x)P(x \mid C)dx}$$

$$= \frac{P(T_o \mid T,C)P(T \mid C)}{normalization}$$

Denominator normalizes so Posterior is PDF.

Combining the Prior Estimate and Observation

$$P(T \mid T_o, C) = \frac{P(T_o \mid T, C)\,P(T \mid C)}{normalization}$$



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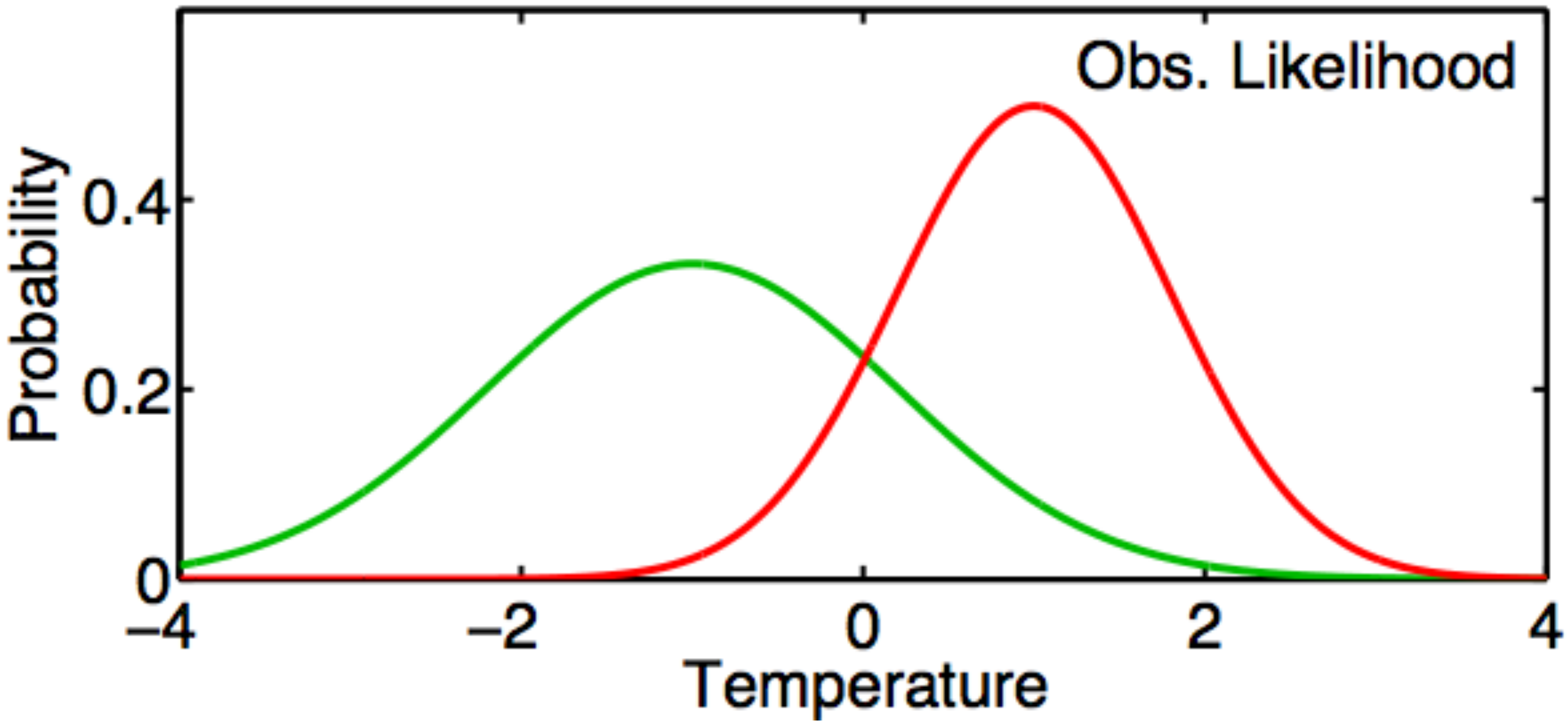

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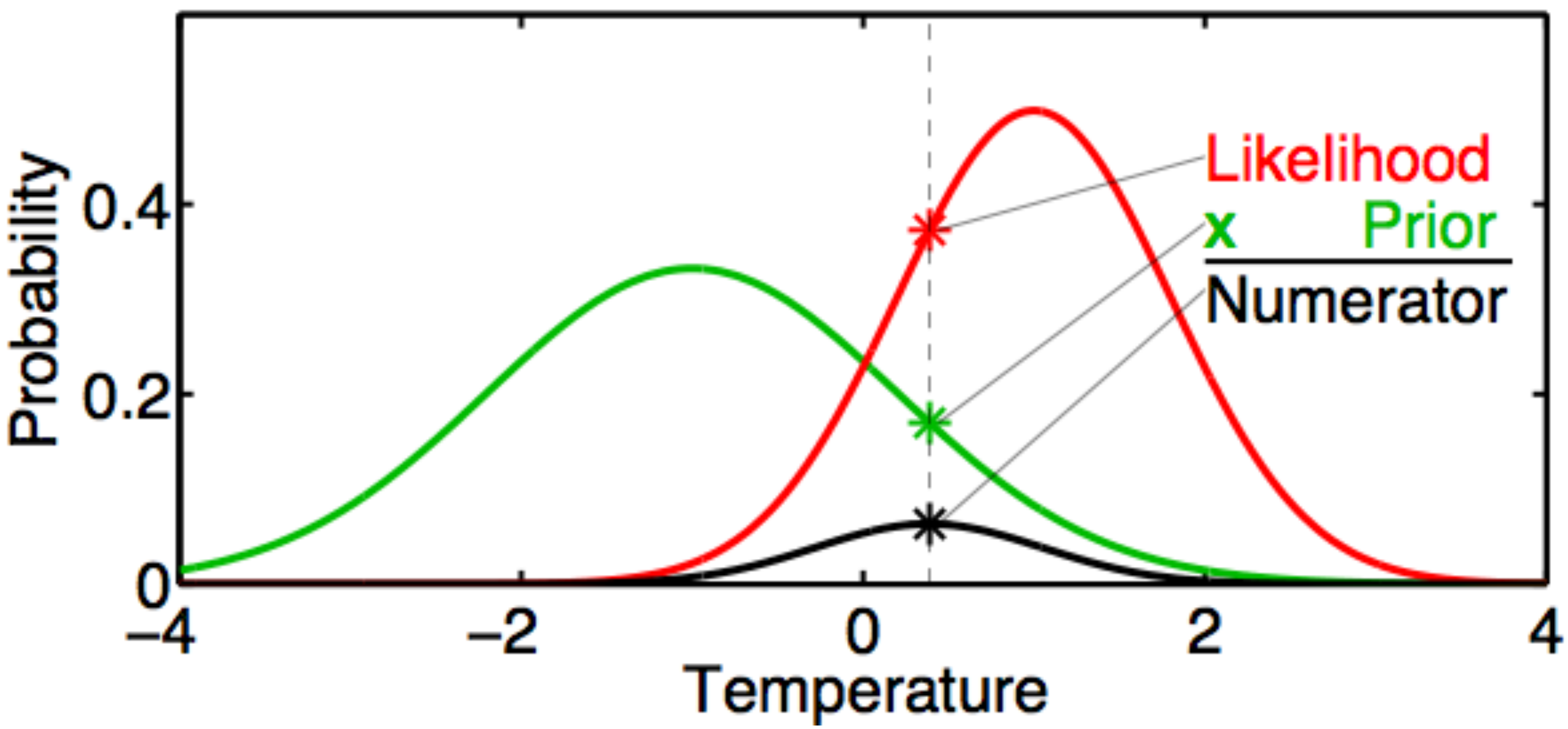

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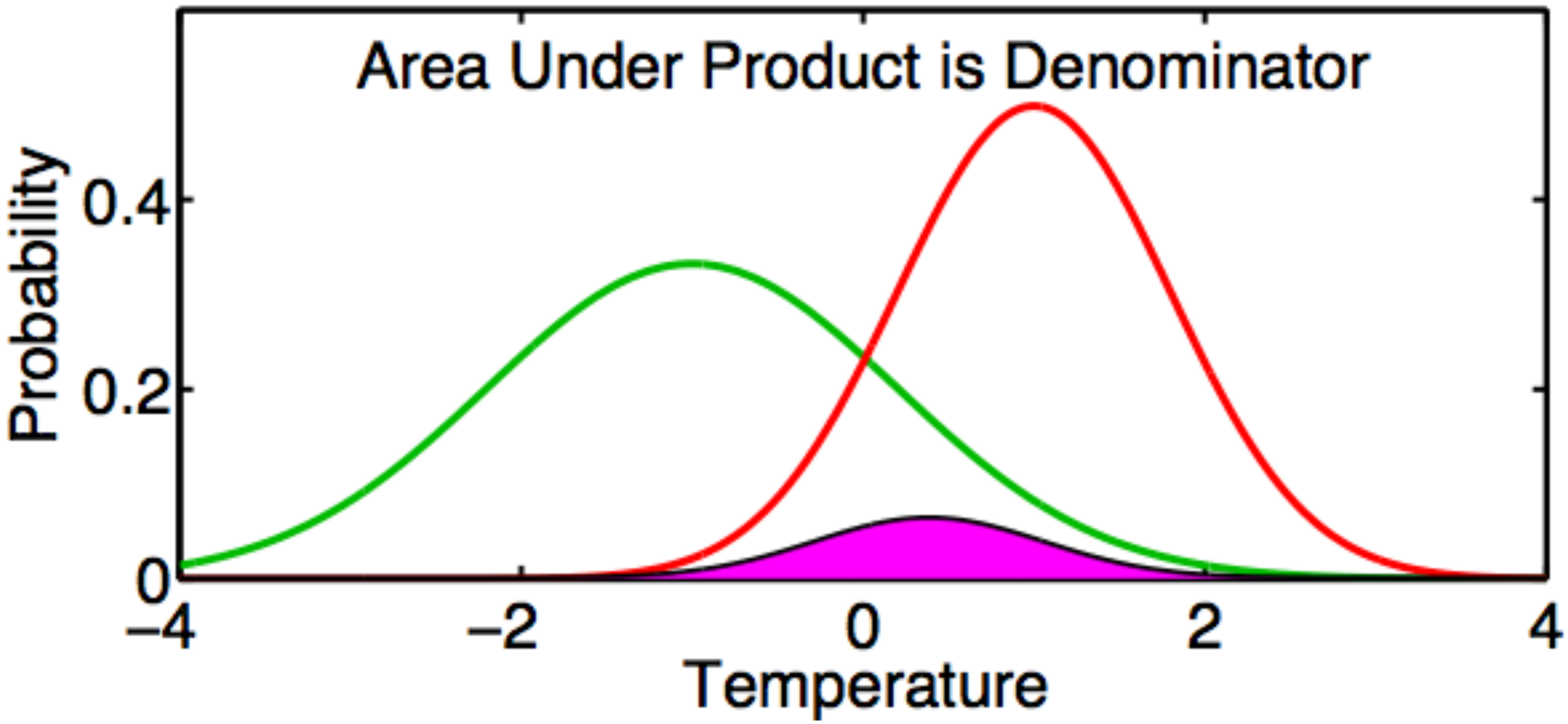

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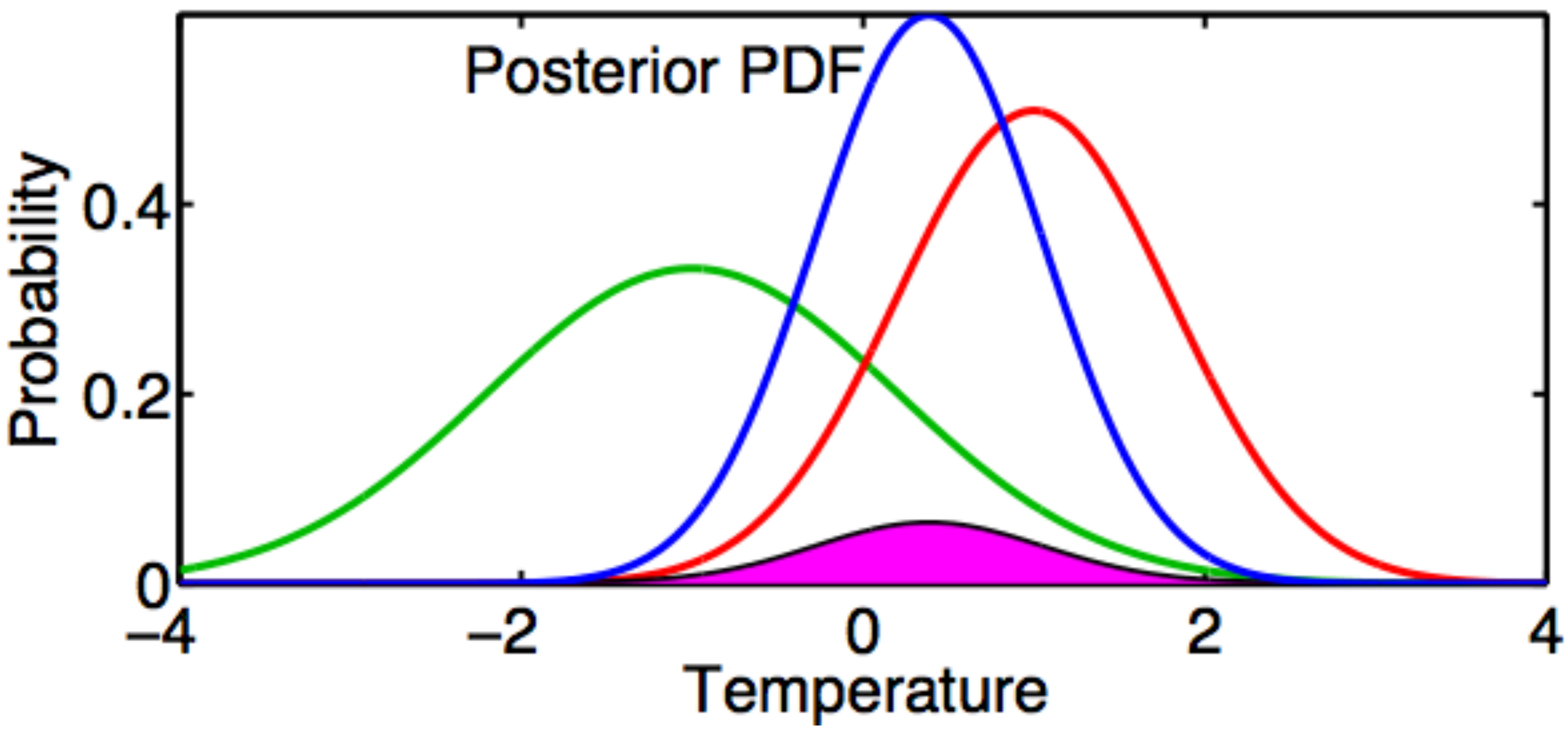


Consistent Color Scheme Throughout Tutorial

Green = Prior

Red = Observation

Blue = Posterior

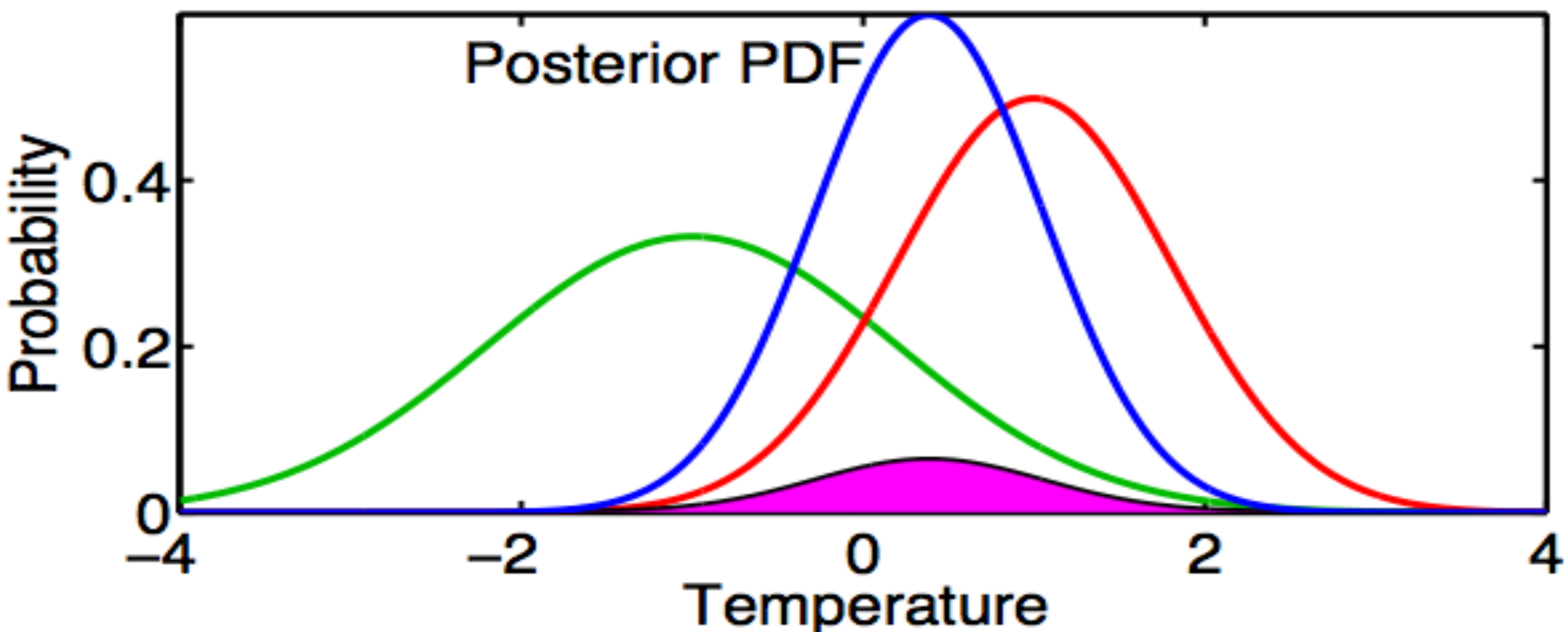
Black = Truth

(truth available only for ‘perfect model’ examples)

Combining the Prior Estimate and Observation

$$P(T \mid T_o, C) = \frac{P(T_o \mid T, C)\,P(T \mid C)}{normalization}$$

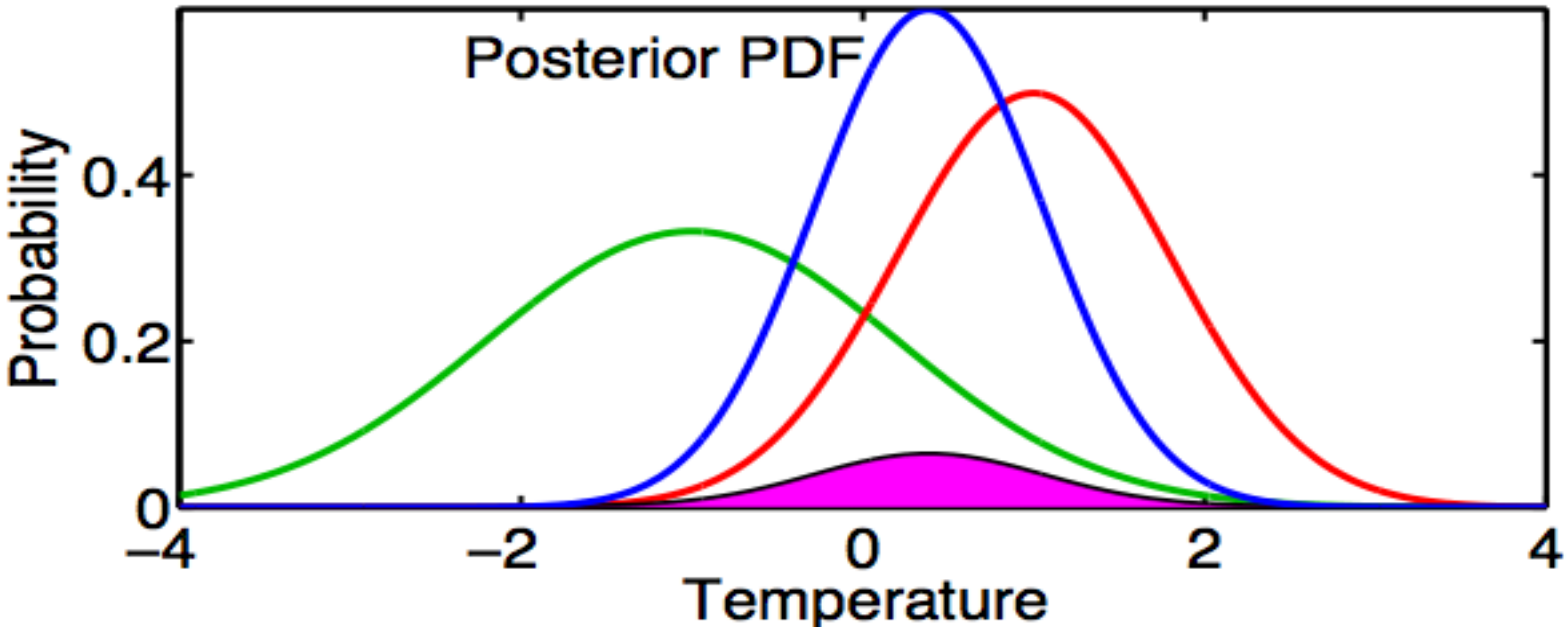
Generally no analytic solution for Posterior.



Combining the Prior Estimate and Observation

$$P(T \mid T_o, C) = \frac{P(T_o \mid T, C)\, P(T \mid C)}{normalization}$$

Gaussian Prior and Likelihood -> Gaussian Posterior



Combining the Prior Estimate and Observation

For Gaussian prior and likelihood…

Prior $P(T \mid C) = Normal(T_p, \sigma_p)$

Likelihood $P(T_o \mid T, C) = Normal(T_o, \sigma_o)$

Then, Posterior $P(T \mid T_o, C) = Normal(T_u, \sigma_u)$

With

$$\sigma_u = \sqrt{\left(\sigma_p^{-2} + \sigma_o^{-2}\right)^{-1}}$$

$$T_u = \sigma_u^2\left[\sigma_p^{-2} T_p + \sigma_o^{-2} T_o\right]$$

The One-Dimensional Kalman Filter

1. Suppose we have a linear forecast model L

   A. If temperature at time $t_1 = T_1$, then
      temperature at $t_2 = t_1 + \Delta t$ is $T_2 = L(T_1)$

   B. Example: $T_2 = T_1 + \Delta t T_1$

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   B. Example: $T_2 = T_1 + \Delta t T_1$.

2. If posterior estimate at time t_1 is *Normal*($T_{u,1}$, $\sigma_{u,1}$) then prior at t_2 is *Normal*($T_{p,2}$, $\sigma_{p,2}$).

   $T_{p,2} = T_{u,1}, + \Delta t T_{u,1}$

   $\sigma_{p,2} = (\Delta t + 1)\ \sigma_{u,1}$

The One-Dimensional Kalman Filter

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3. Given an observation at t_2 with distribution *Normal*(t_o, σ_o) the likelihood is also *Normal*(t_o, σ_o).

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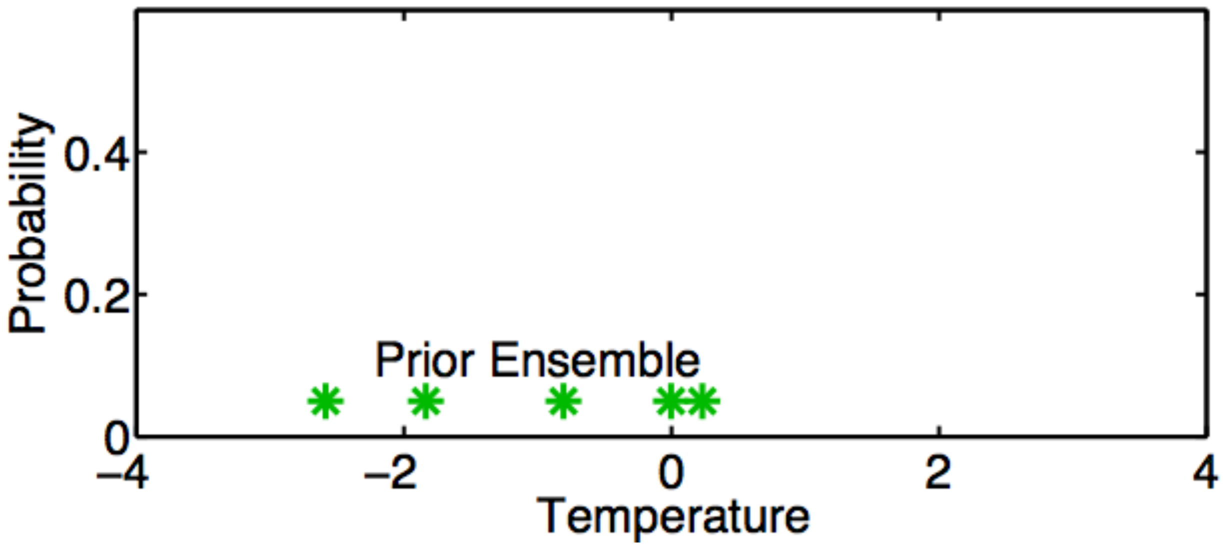
4. The posterior at t_2 is *Normal*$(T_{u,2}, \sigma_{u,2})$ where $T_{u,2}$ and $\sigma_{u,2}$ come from page 19.

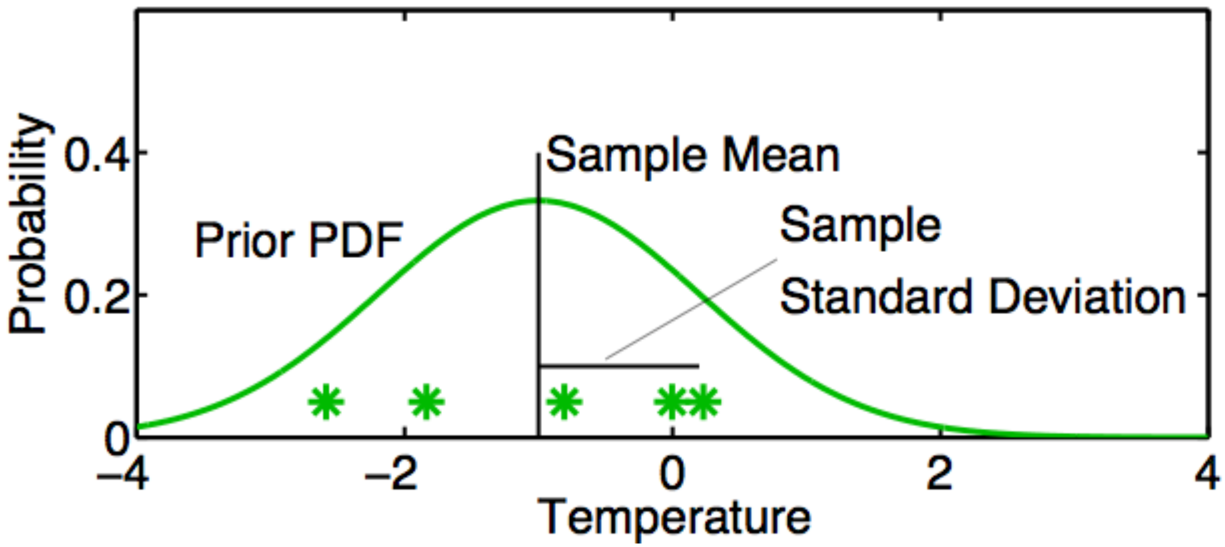
A One-Dimensional Ensemble Kalman Filter

Represent a prior pdf by a sample (ensemble) of N values:



A One-Dimensional Ensemble Kalman Filter

Represent a prior pdf by a sample (ensemble) of N values:



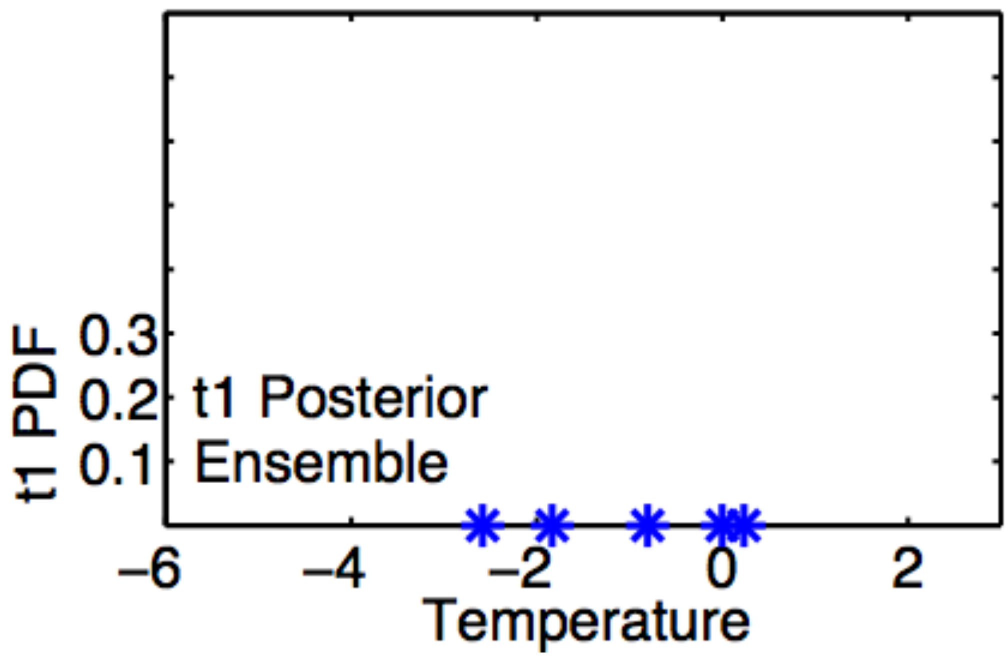
Use sample mean $\overline{T} = \sum_{n=1}^{N} T_n / N$

and sample standard deviation $\sigma_T = \sqrt{\sum_{n=1}^{N} \left(T_n - \overline{T}\right)^2 / (N-1)}$

to determine a corresponding continuous distribution $Normal\left(\overline{T}, \sigma_T\right)$

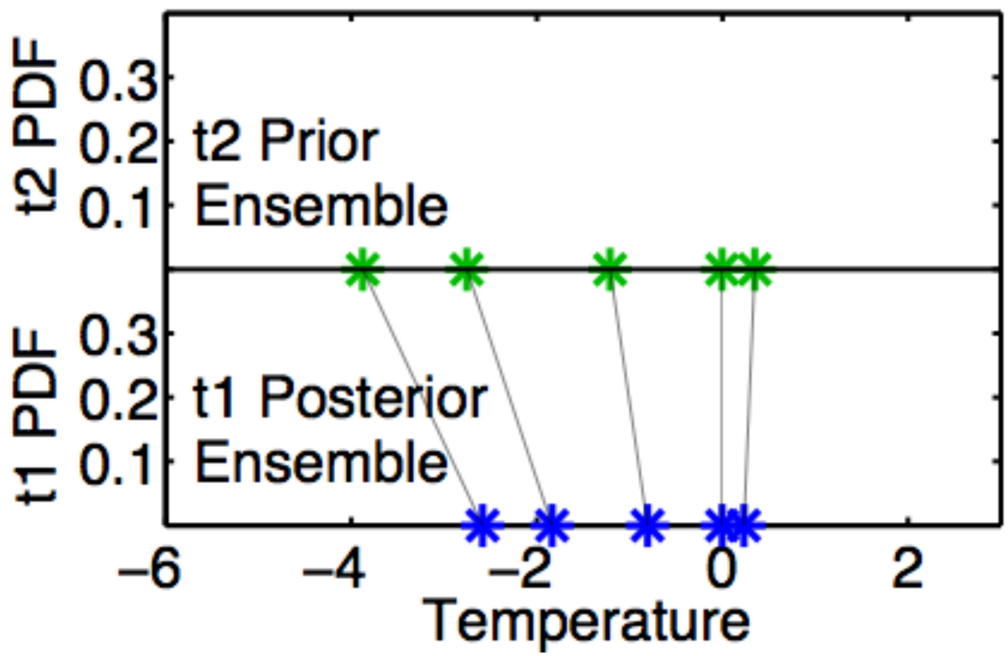
A One-Dimensional Ensemble Kalman Filter: Model Advance

If posterior ensemble at time t_1 is $T_{1,n}$, $n = 1, \ldots, N$



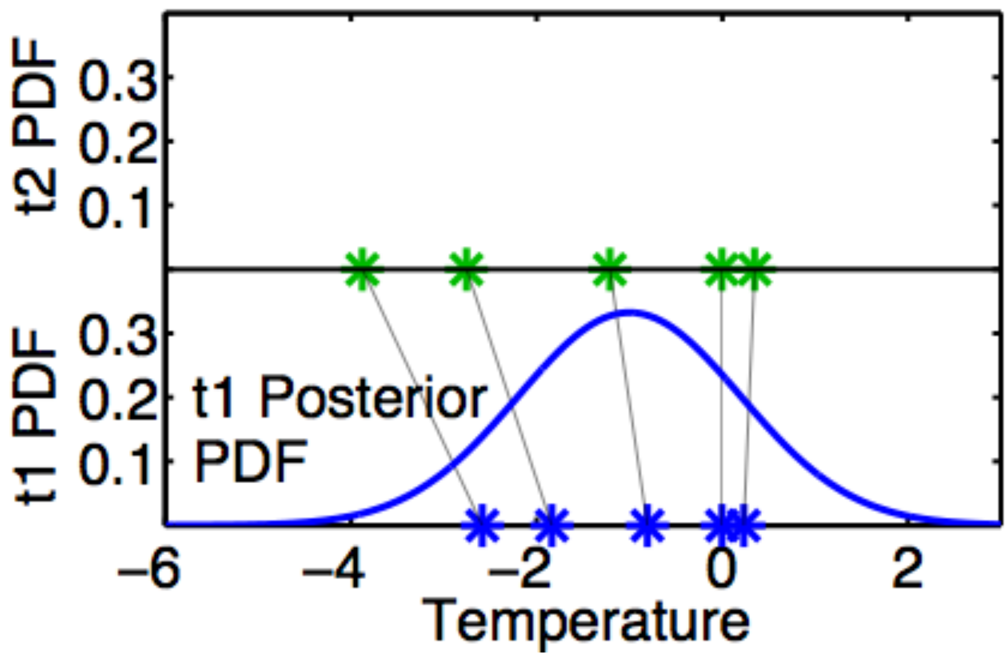
A One-Dimensional Ensemble Kalman Filter: Model Advance

If posterior ensemble at time t_1 is $T_{1,n}$, $n = 1, \dots N$,
advance each member to time t_2 with model, $T_{2,n} = L(T_{1,n})$ $n = 1, \dots, N$.



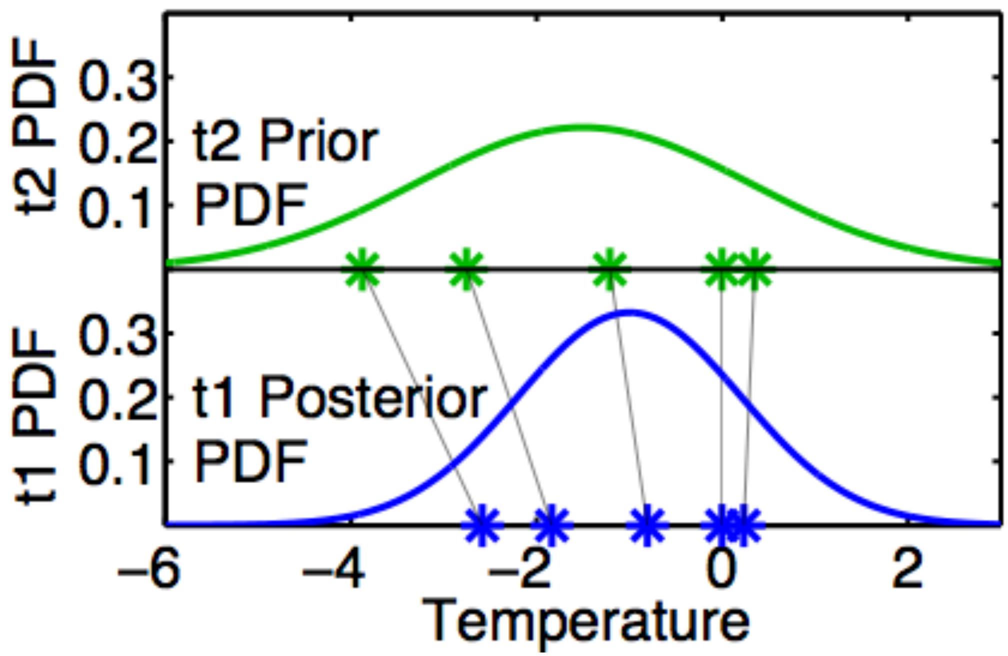
A One-Dimensional Ensemble Kalman Filter: Model Advance

Same as advancing continuous pdf at time t_1 …

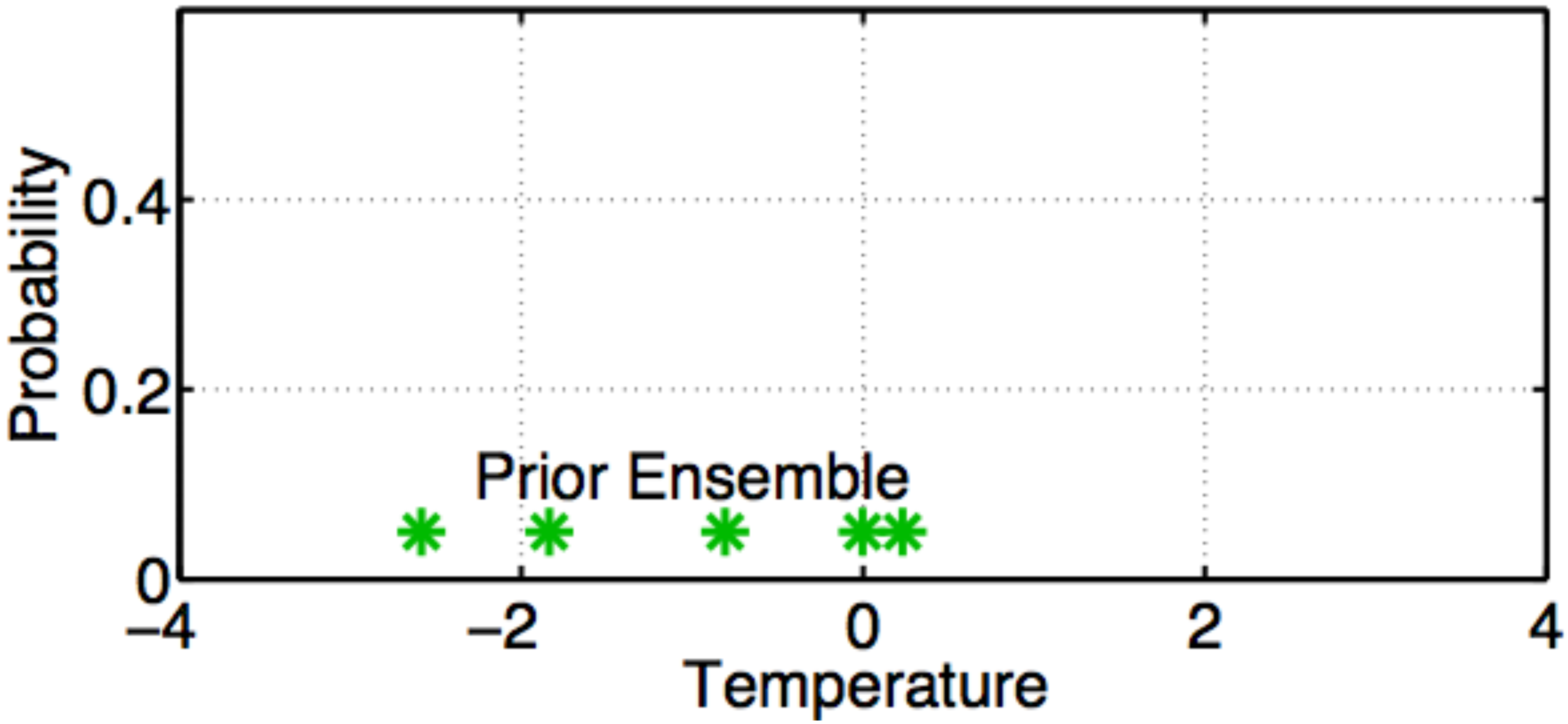


A One-Dimensional Ensemble Kalman Filter: Model Advance

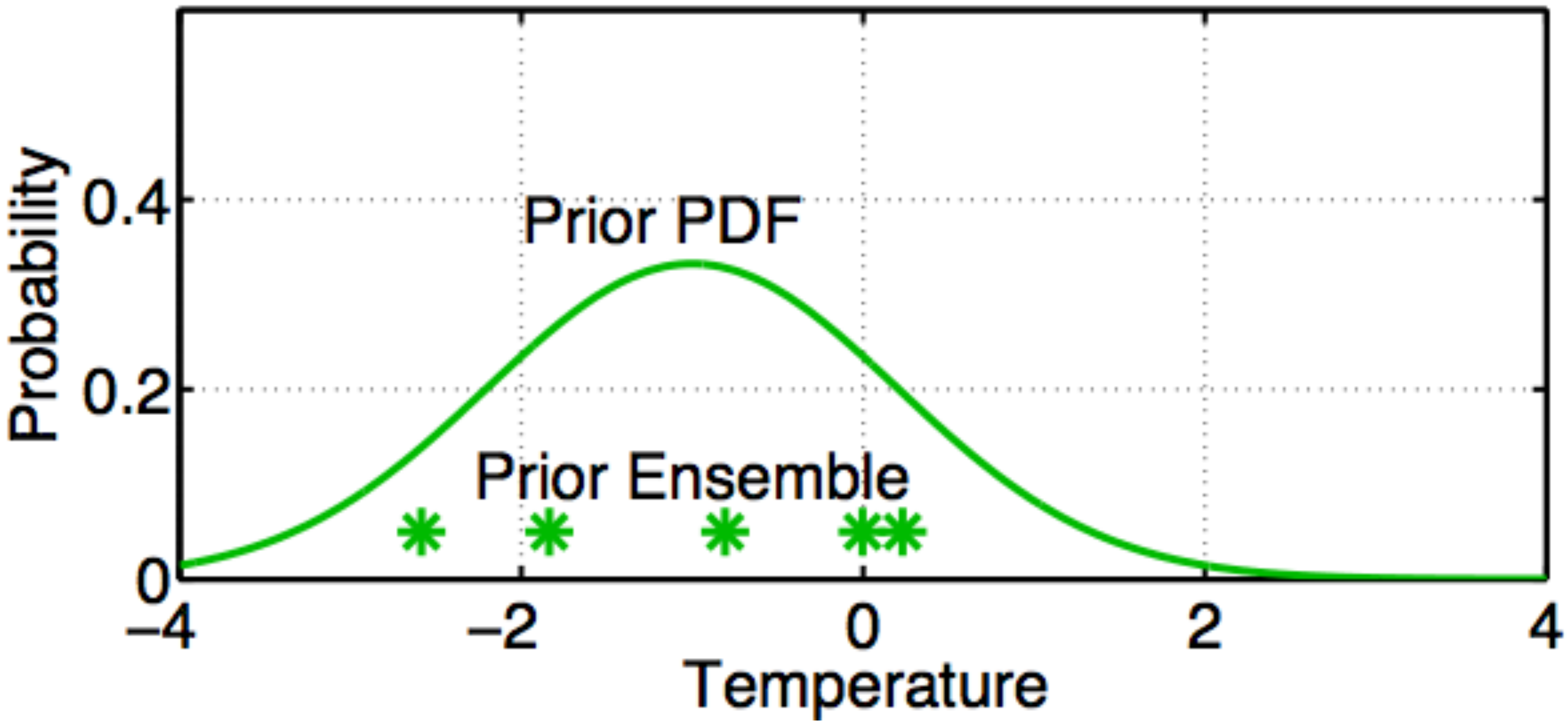
Same as advancing continuous pdf at time t_1 to time t_2 with model L.



A One-Dimensional Ensemble Kalman Filter: Assimilating an Observation

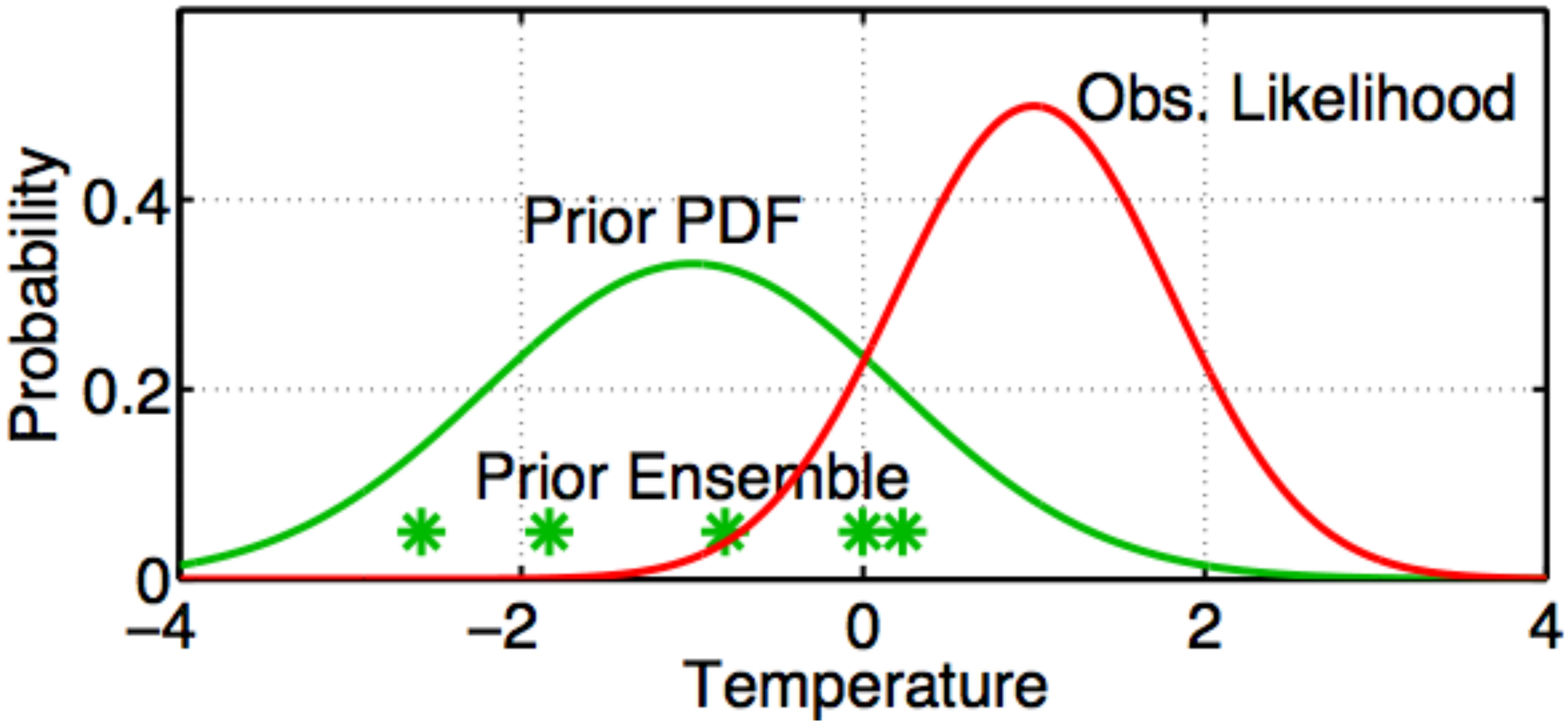


A One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



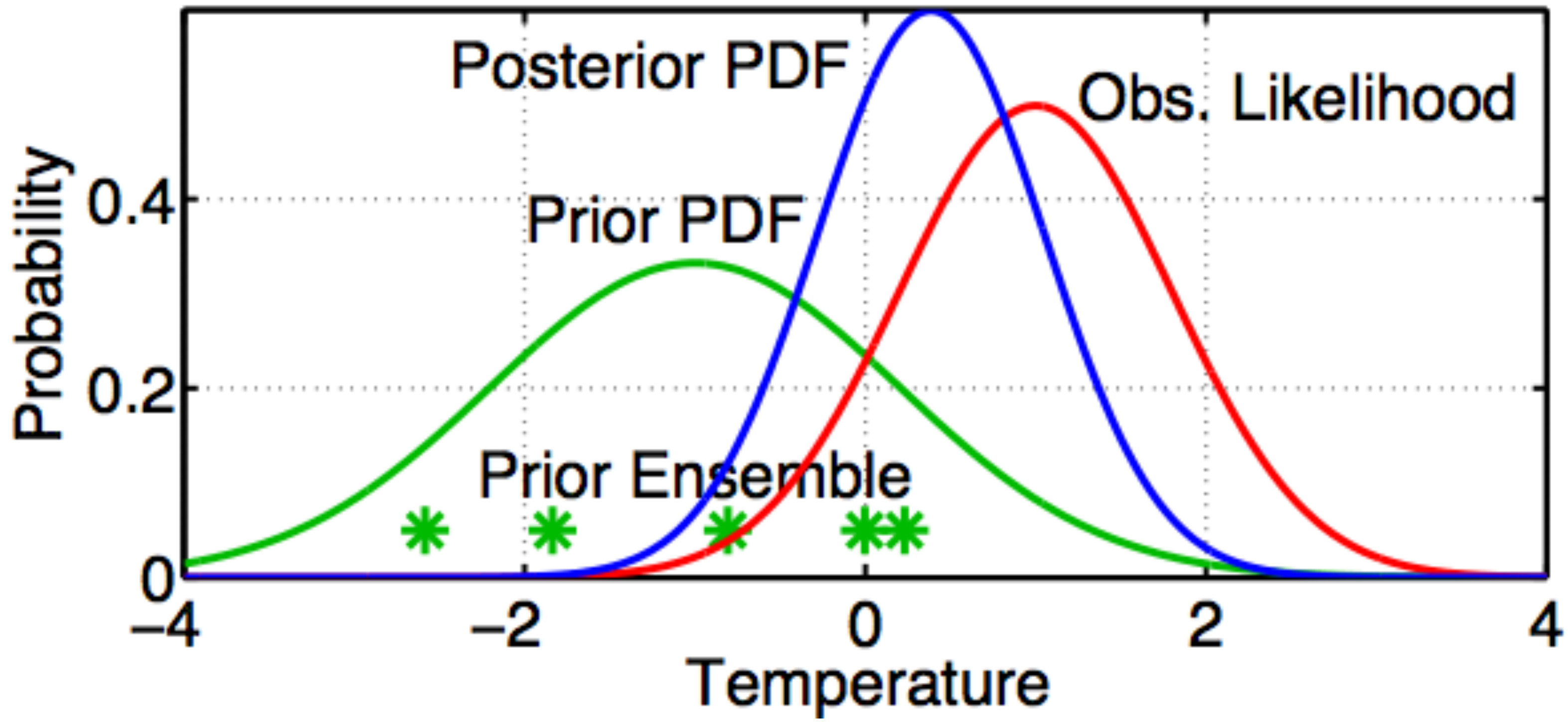
Fit a Gaussian to the sample.

A One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



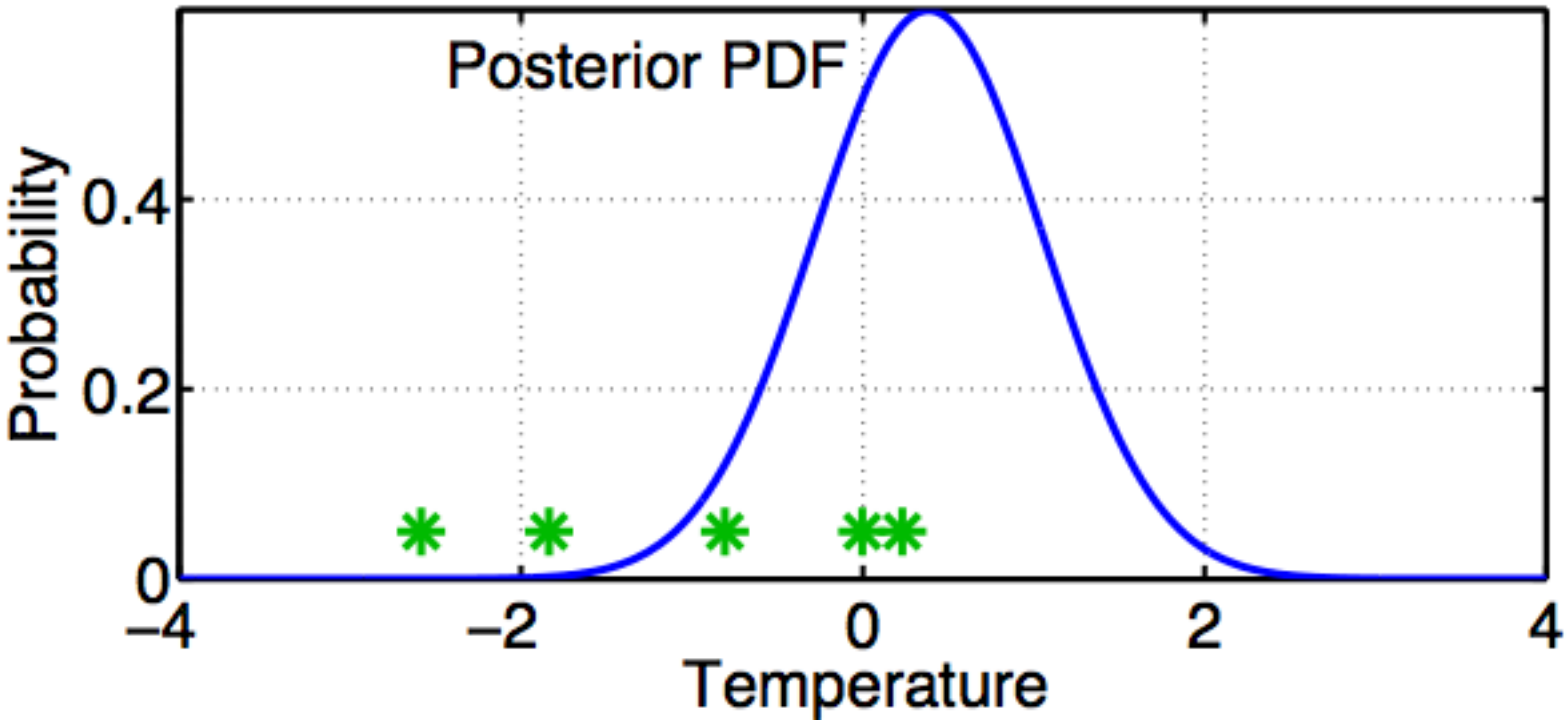
Get the observation likelihood.

A One-Dimensional Ensemble Kalman Filter:
Assimilating an Observation



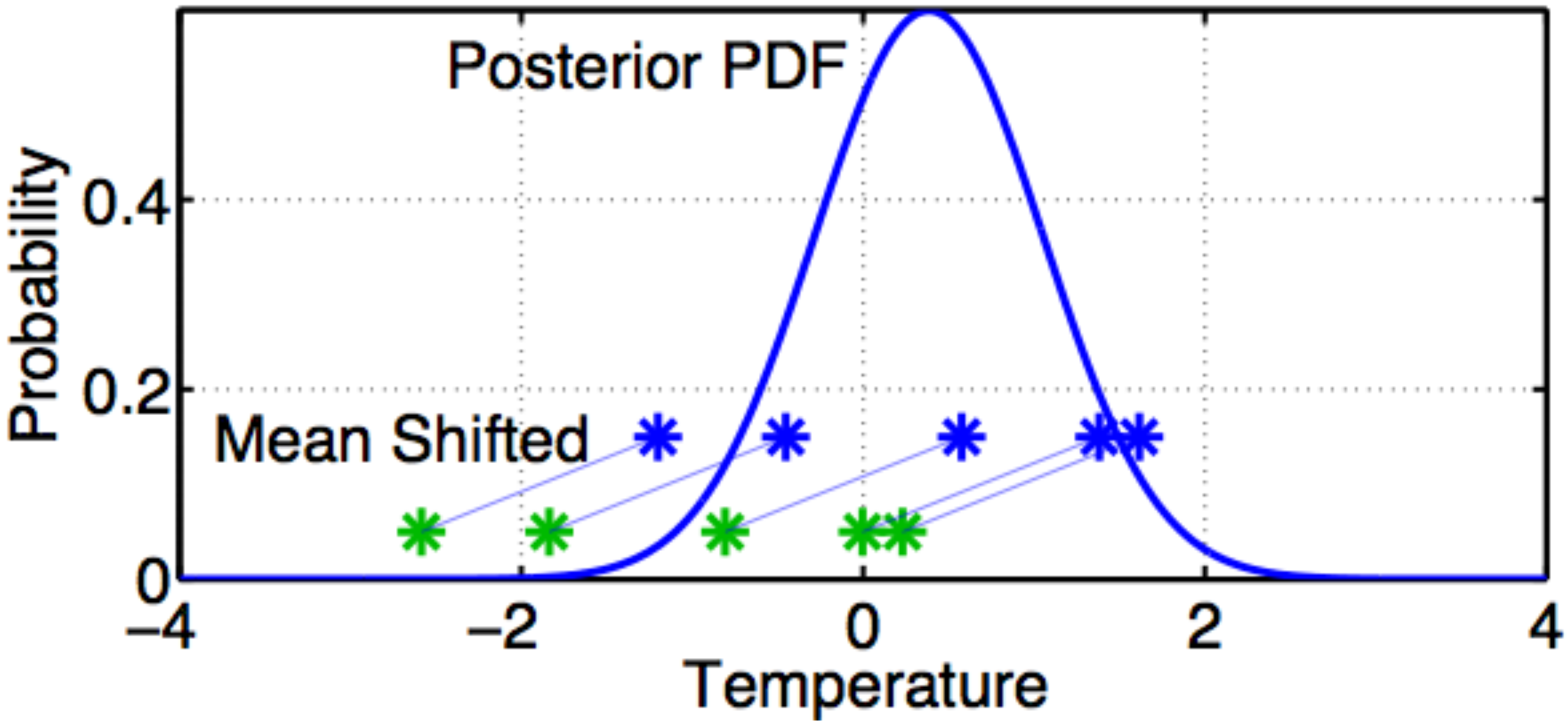
Compute the continuous posterior PDF.

A One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



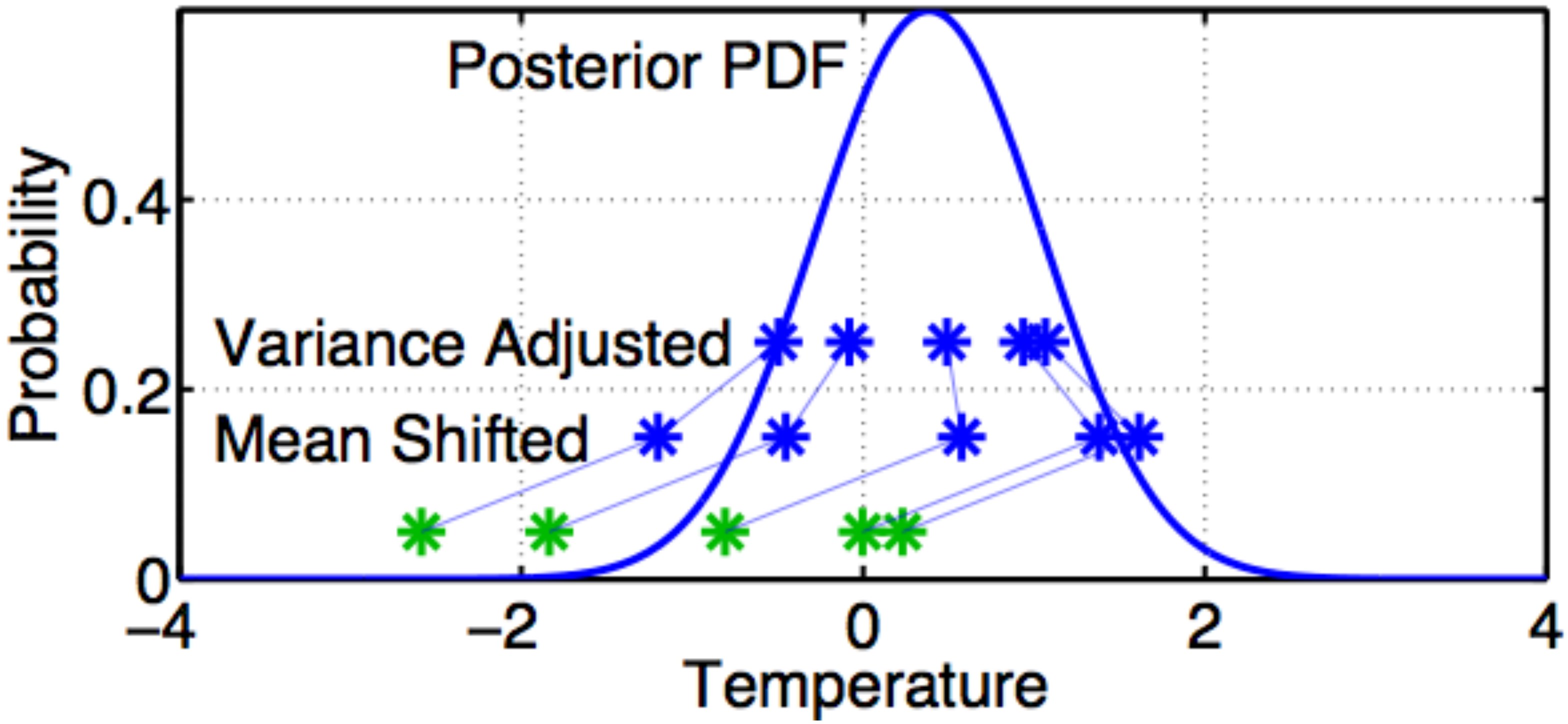
Use a deterministic algorithm to 'adjust' the ensemble.

A One-Dimensional Ensemble Kalman Filter:
Assimilating an Observation



First, ‘shift’ the ensemble to have the exact mean of the posterior.

A One-Dimensional Ensemble Kalman Filter: Assimilating an Observation

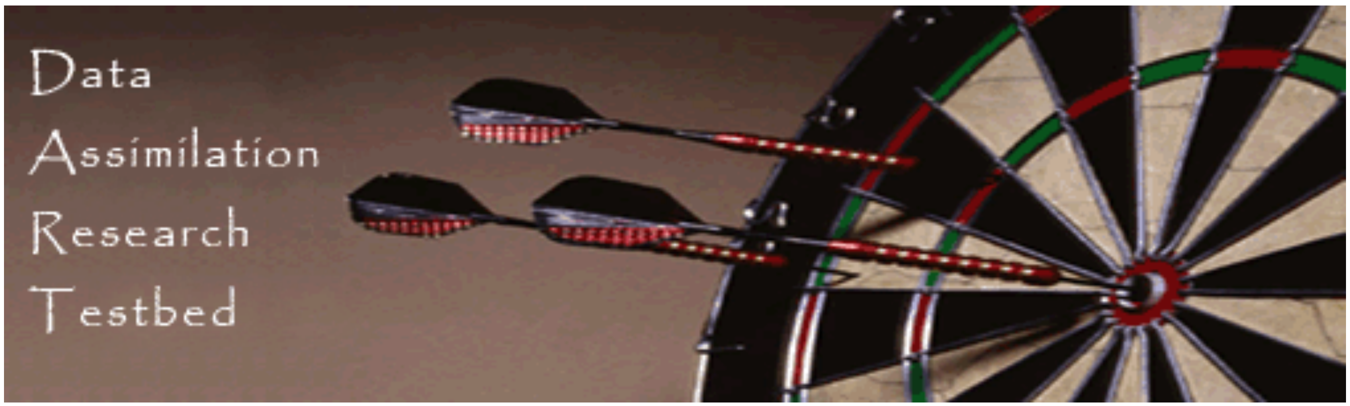


First, ‘shift’ the ensemble to have the exact mean of the posterior.

Second, linearly contract to have the exact variance of the posterior.

Sample statistics are identical to Kalman filter.

We now know how to assimilate a single observed variable.



Section 2: How should observations of one state variable impact an unobserved state variable?

Single observed variable, single unobserved variable

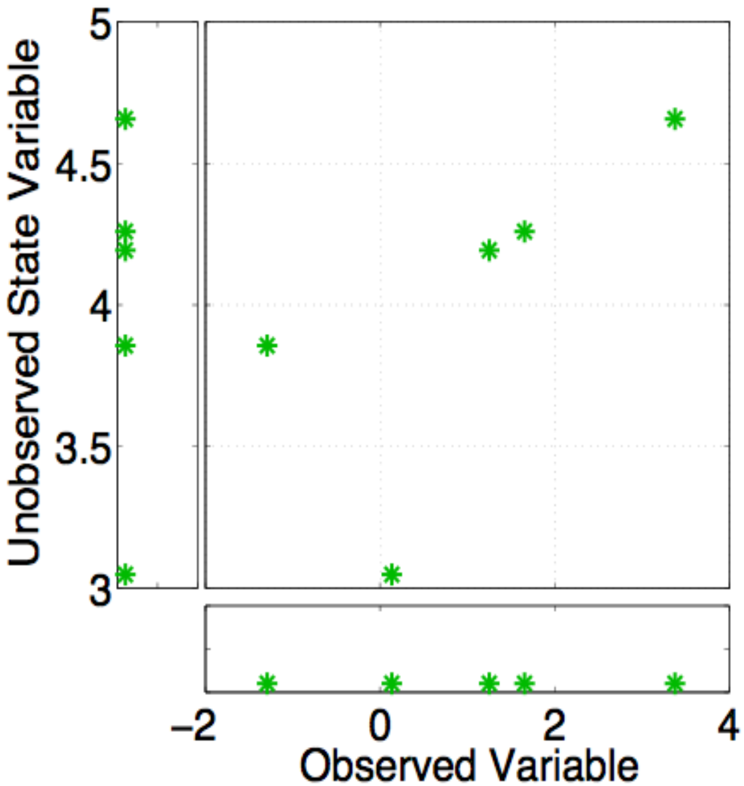
So far, we have a known observation likelihood for single variable.

Now, suppose the prior has an additional variable.

Examine how ensemble members update the additional variable.

Basic method generalizes to any number of additional variables.

Ensemble filters: Updating additional prior state variables

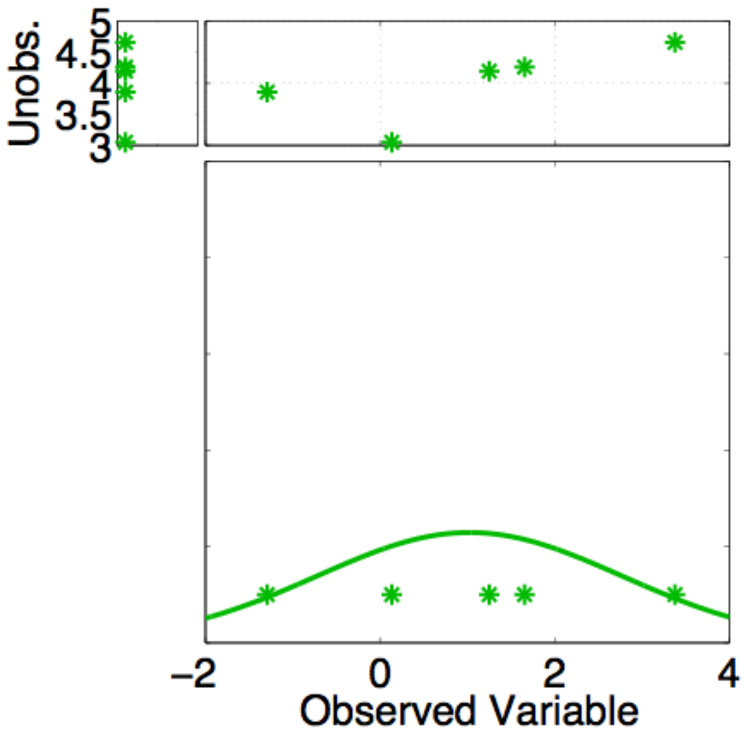


Assume that all we know is prior joint distribution.

One variable is observed, temperature at Boulder.

What should happen to an unobserved variable, like temperature at Denver?

Ensemble filters: Updating additional prior state variables



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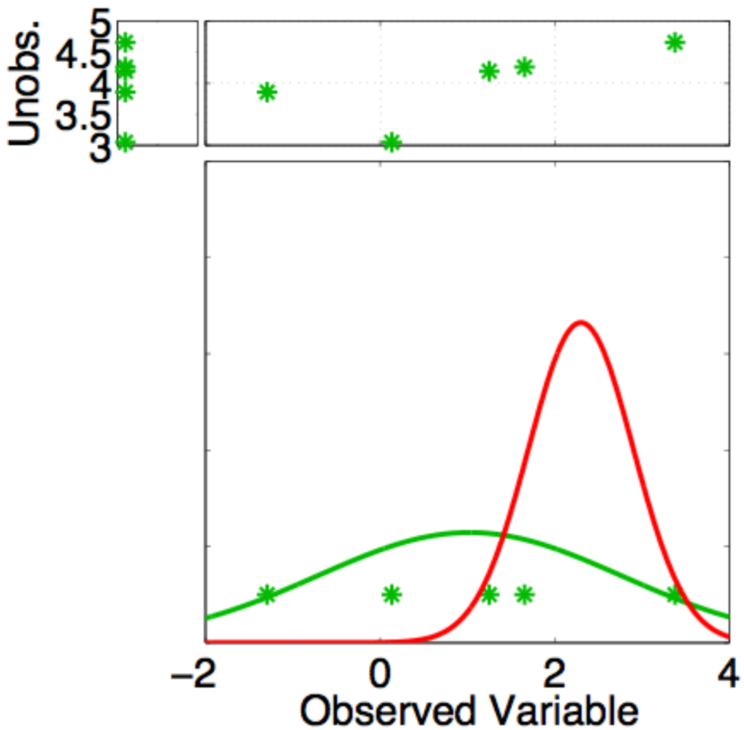
One variable is observed.

Update observed variable as in previous section.

Ensemble filters: Updating additional prior state variables



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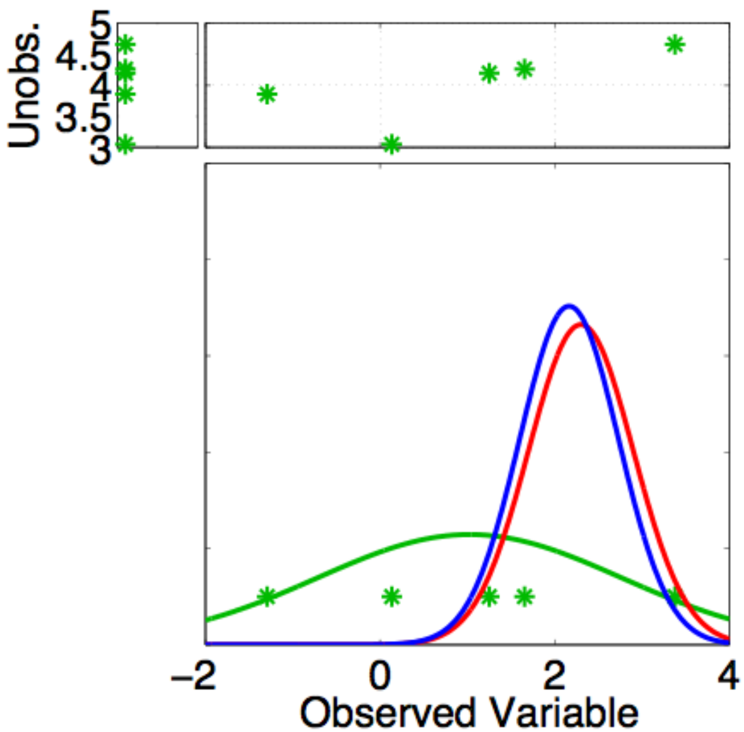

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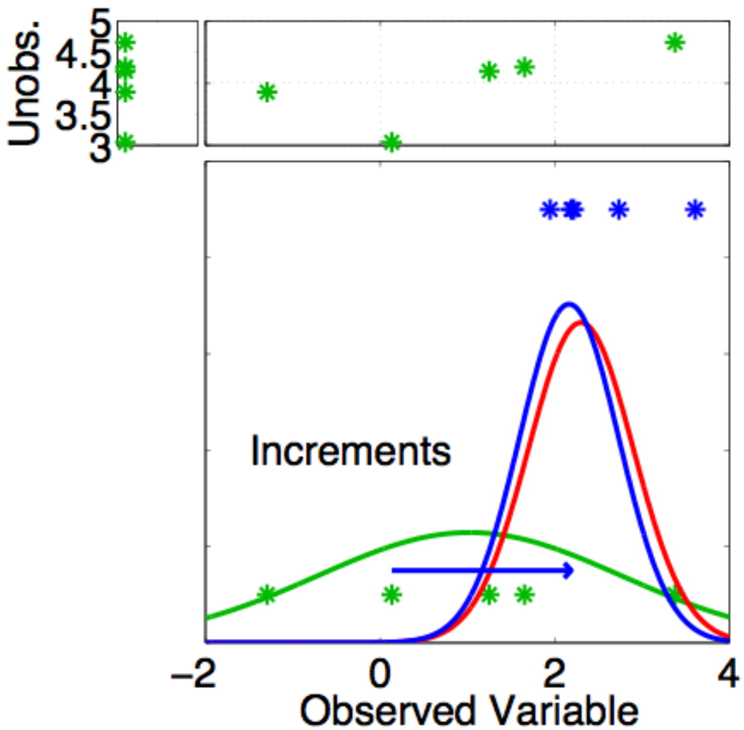
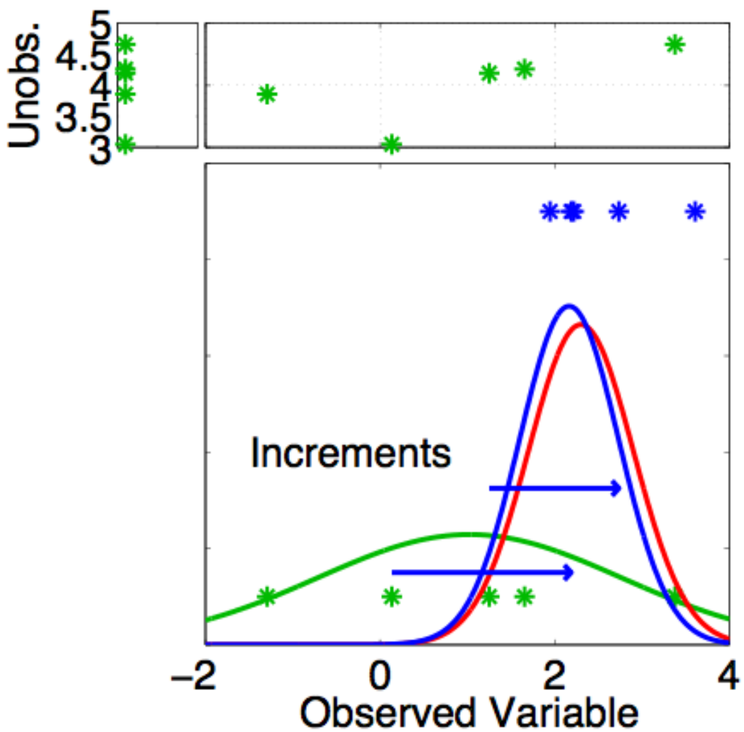


Assume that all we know is prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

Ensemble filters: Updating additional prior state variables



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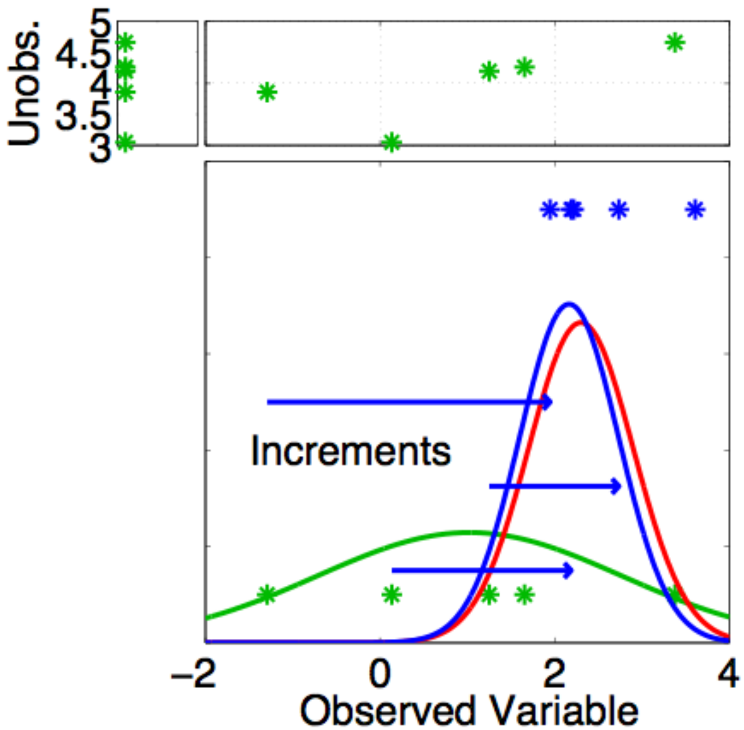

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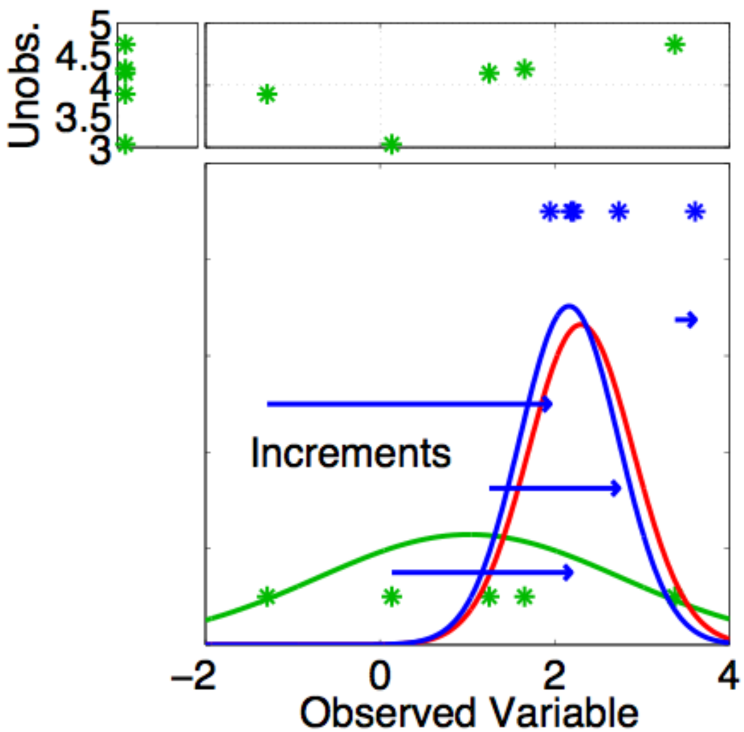

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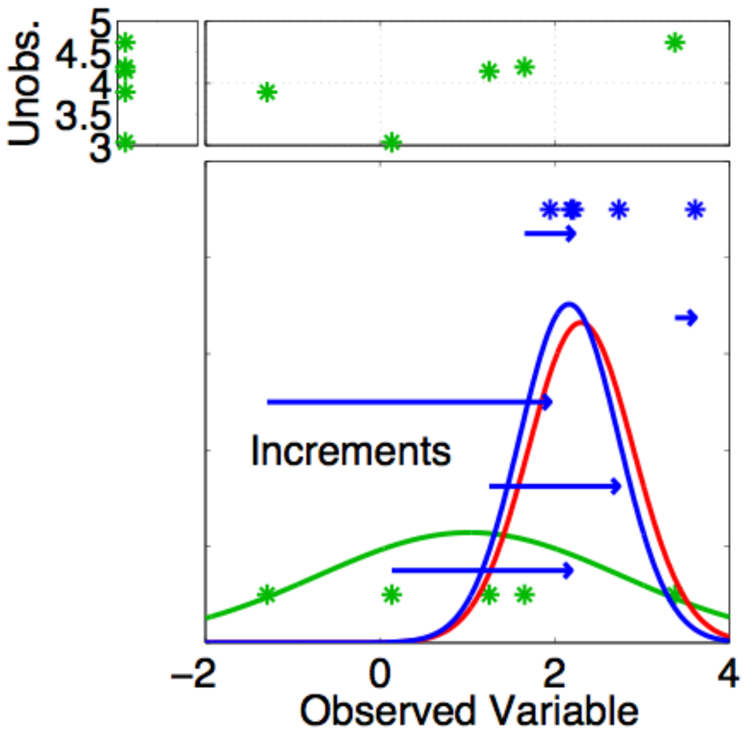

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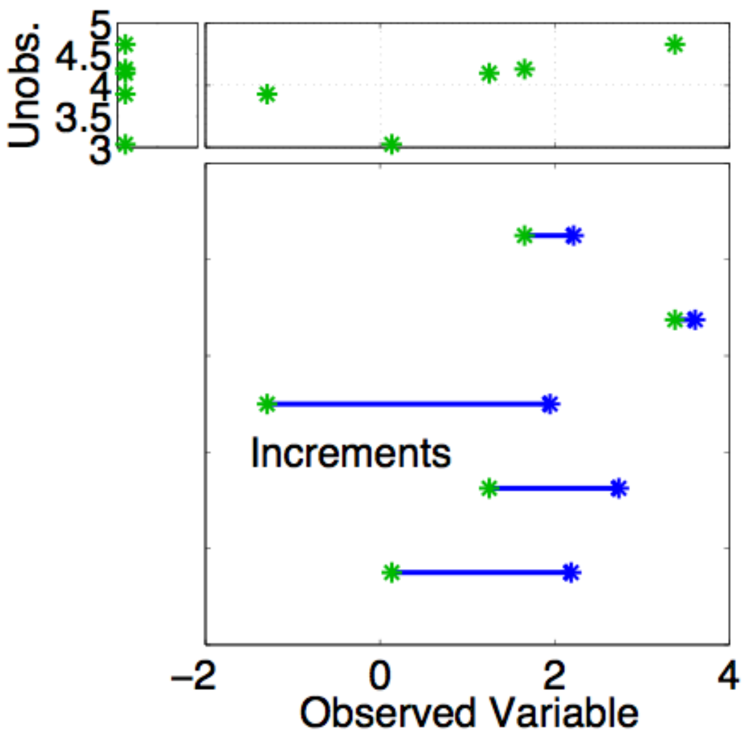

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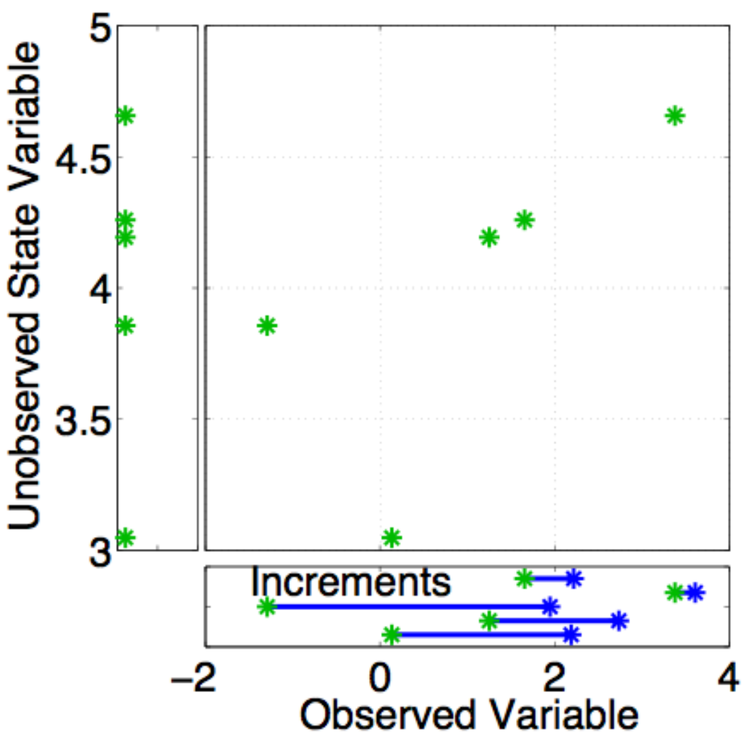
One variable is observed.

Using only increments guarantees that if observation had no impact on observed variable, unobserved variable is unchanged (highly desirable).

Ensemble filters: Updating additional prior state variables



Assume that all we know is prior joint distribution.

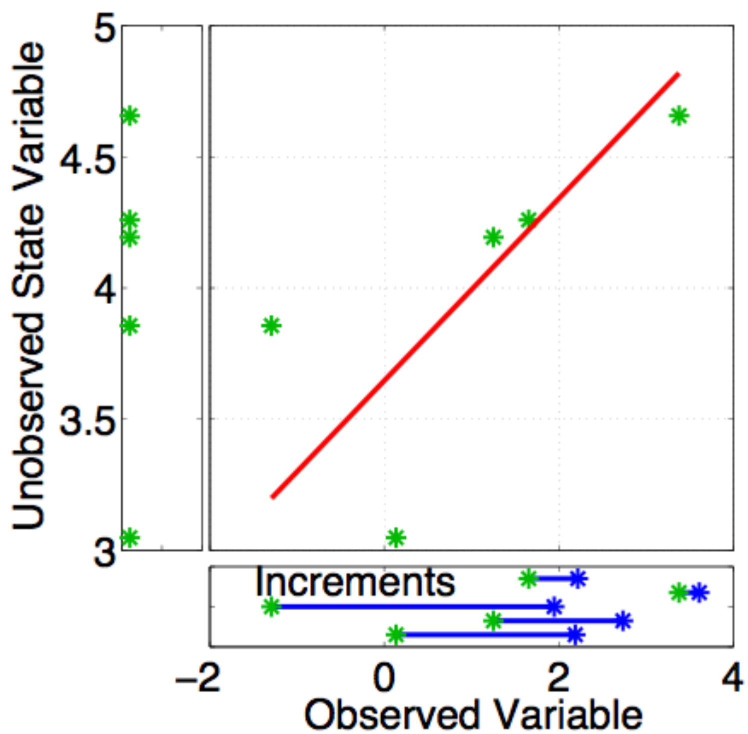
How should the unobserved variable be impacted?

First choice: least squares.

Equivalent to linear regression.

Same as assuming binormal prior.

Ensemble filters: Updating additional prior state variables



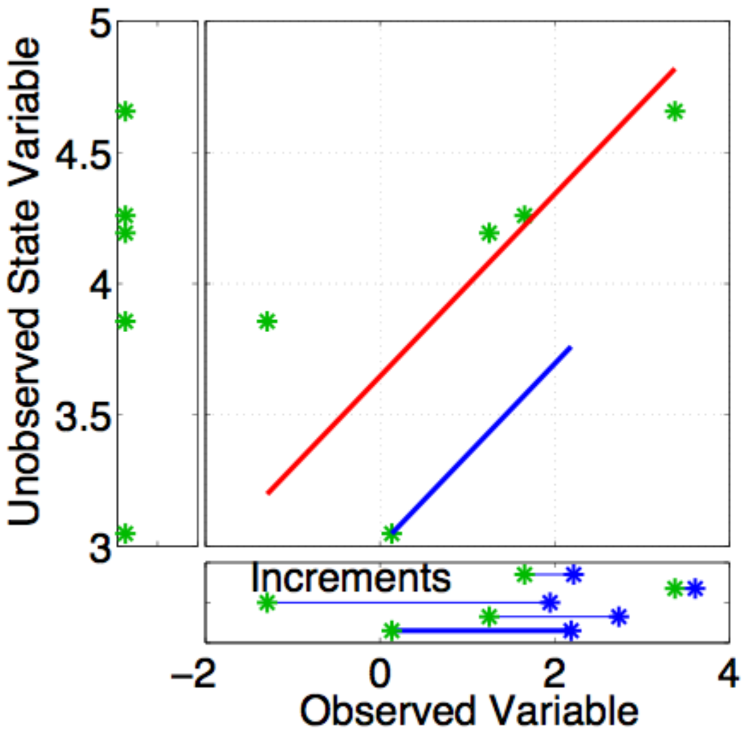
Have joint prior distribution of two variables.

How should the unobserved variable be impacted?

First choice: least squares.

Begin by finding least squares fit.

Ensemble filters: Updating additional prior state variables

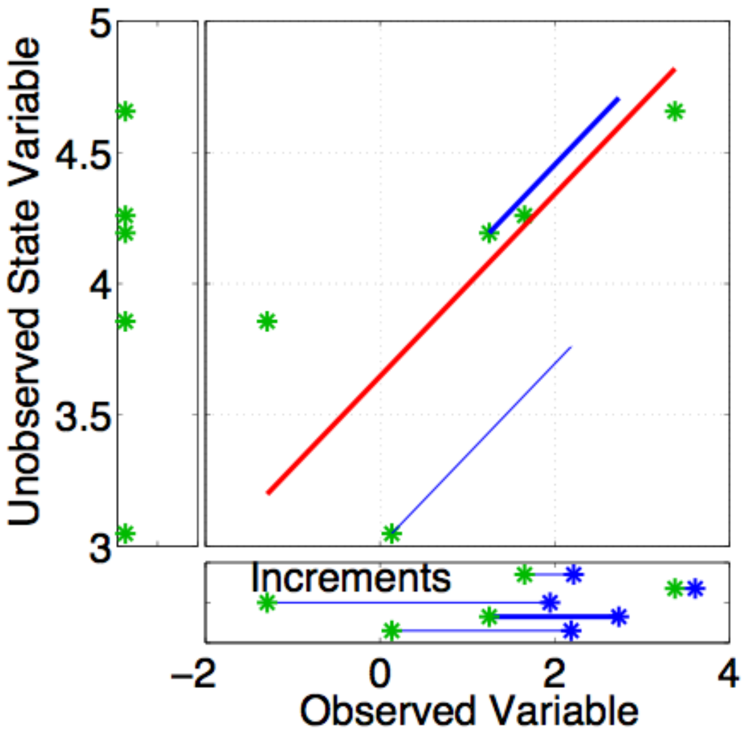


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables



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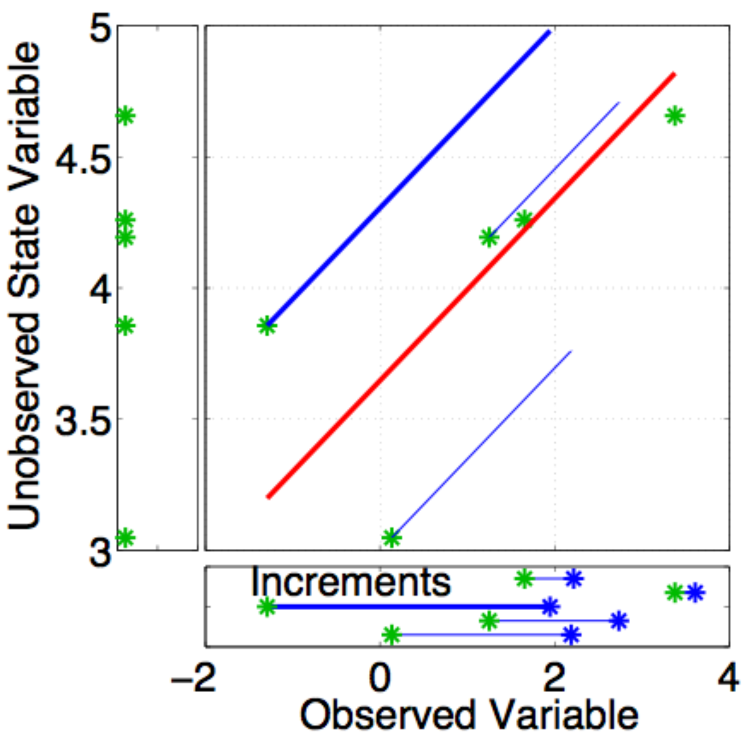

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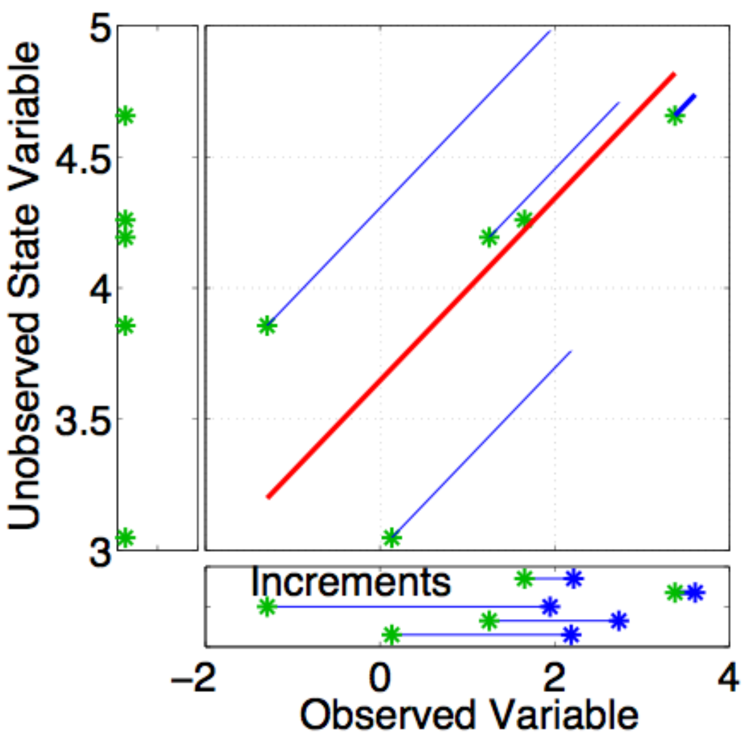
Ensemble filters: Updating additional prior state variables



Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

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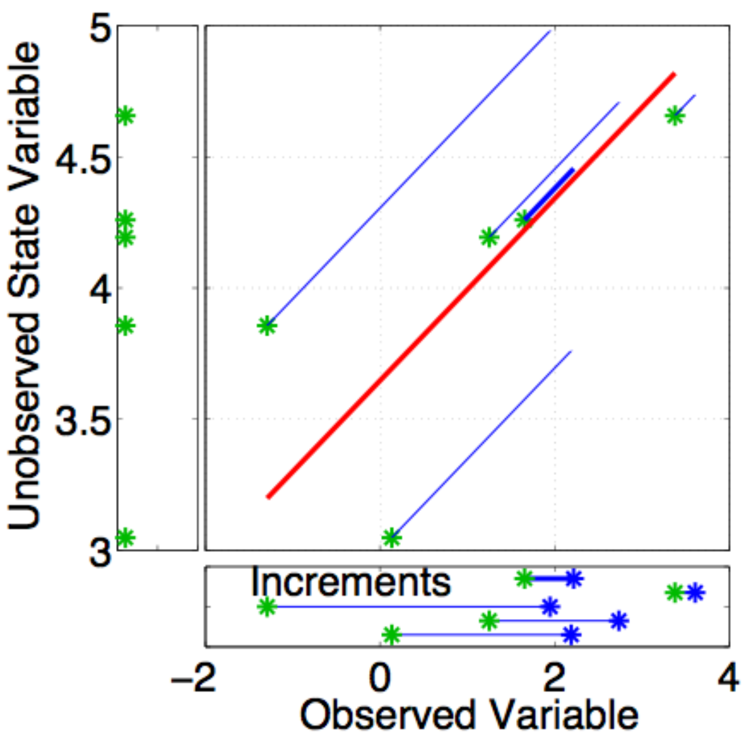

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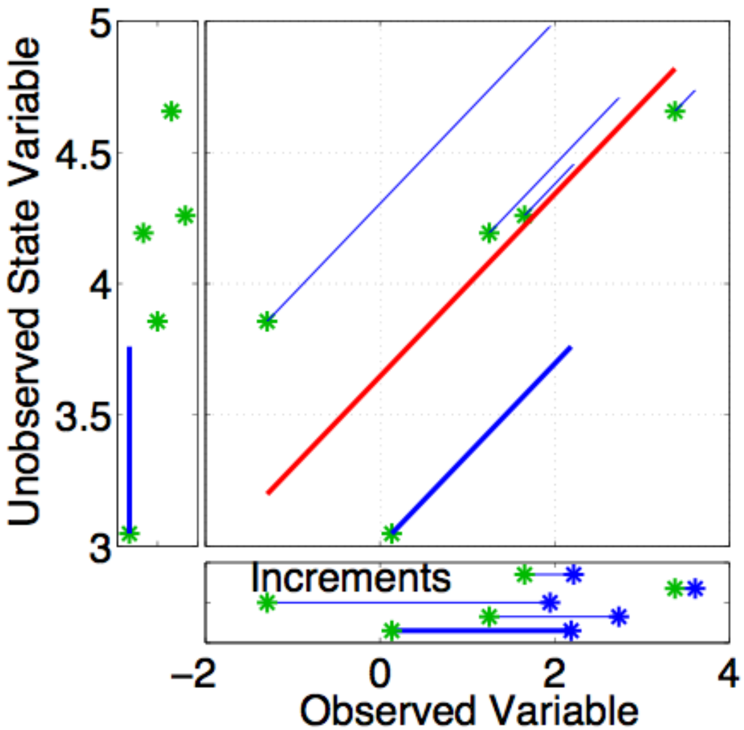

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Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

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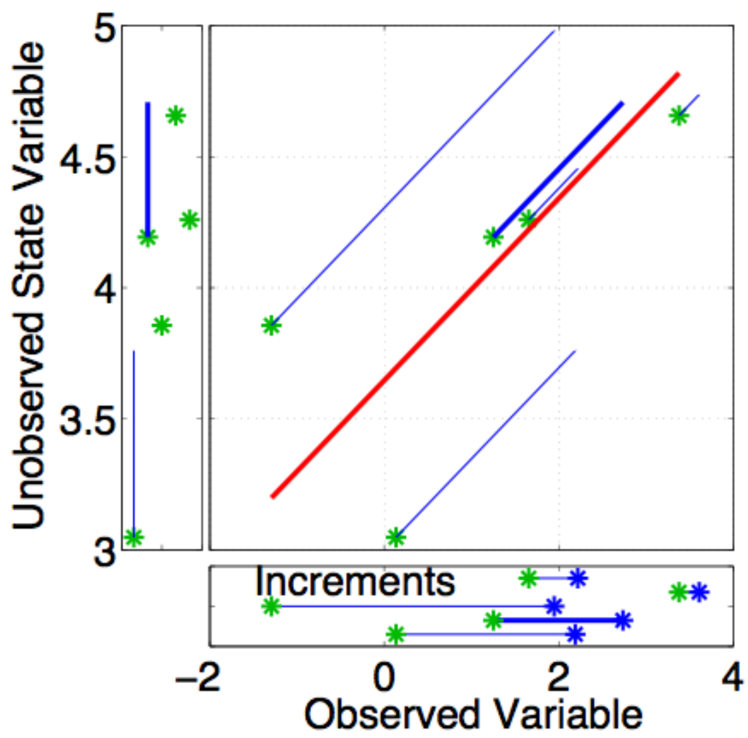

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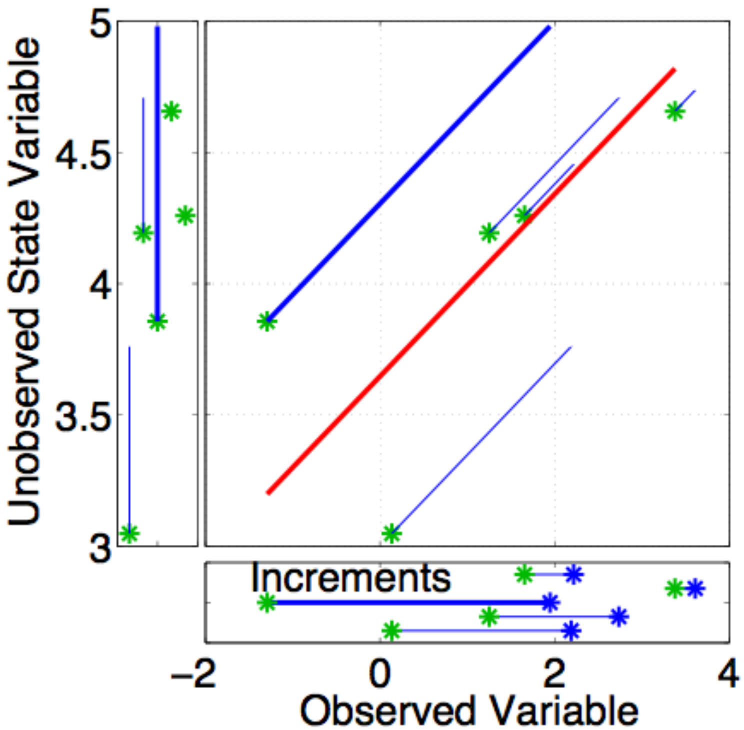

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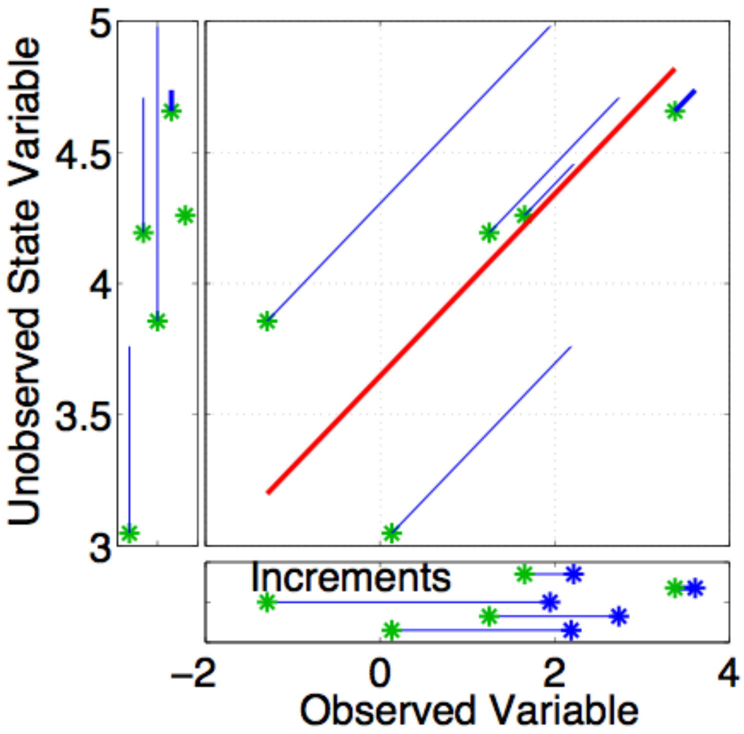

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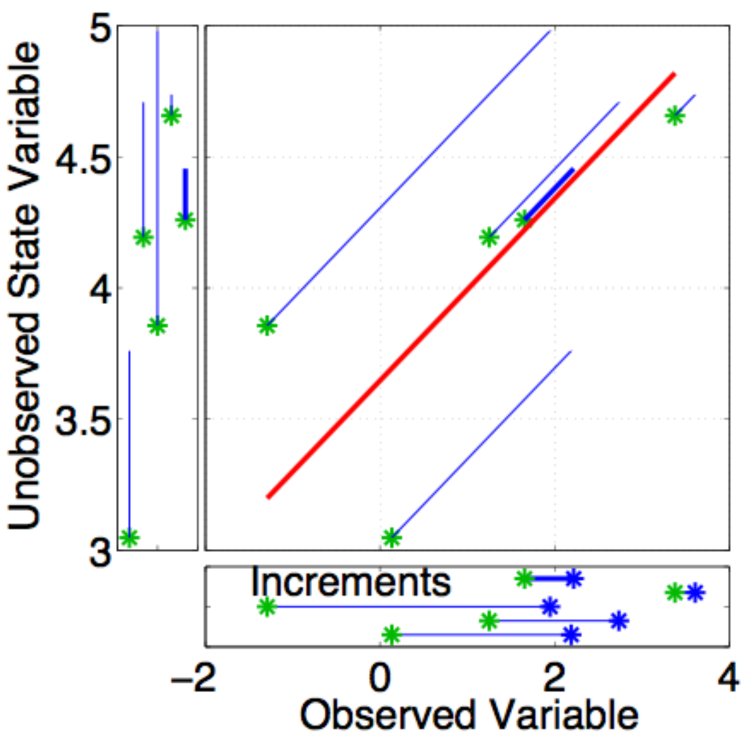

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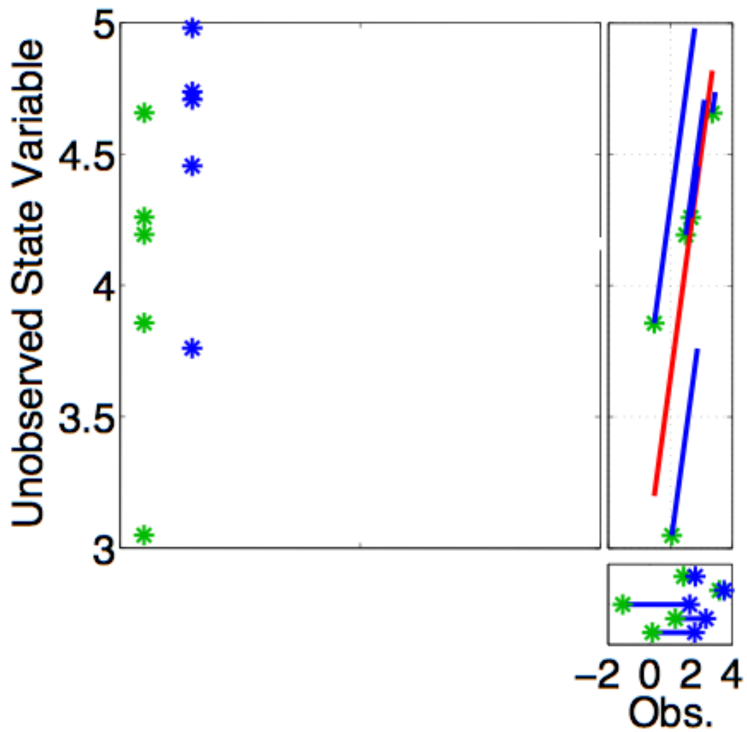
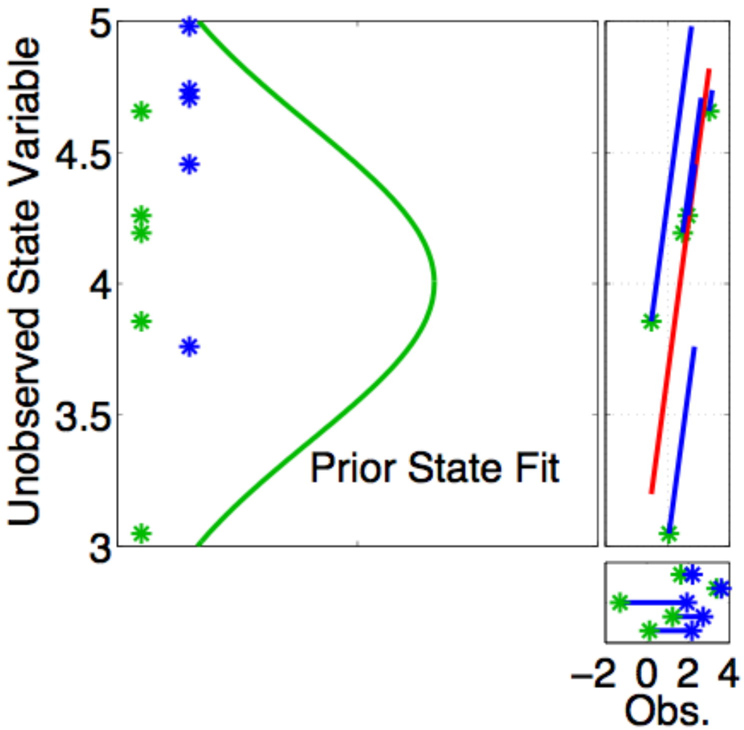


Now have an updated (posterior) ensemble for the unobserved variable.

Ensemble filters: Updating additional prior state variables



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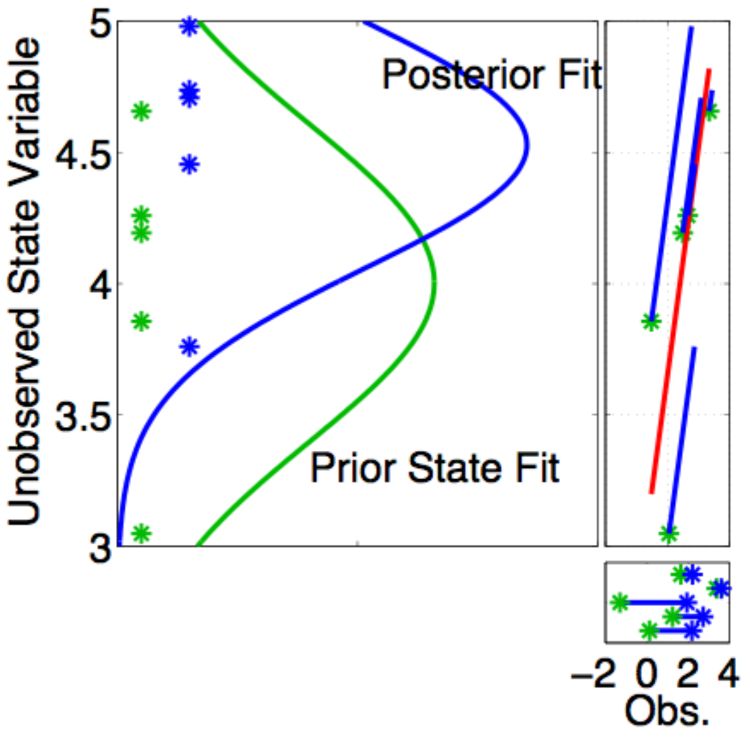
Fitting Gaussians shows that mean and variance have changed.

Ensemble filters: Updating additional prior state variables



Now have an updated (posterior) ensemble for the unobserved variable.

Fitting Gaussians shows that mean and variance have changed.

Other features of the prior distribution may also have changed.

Ensemble Filter for Large Geophysical Models

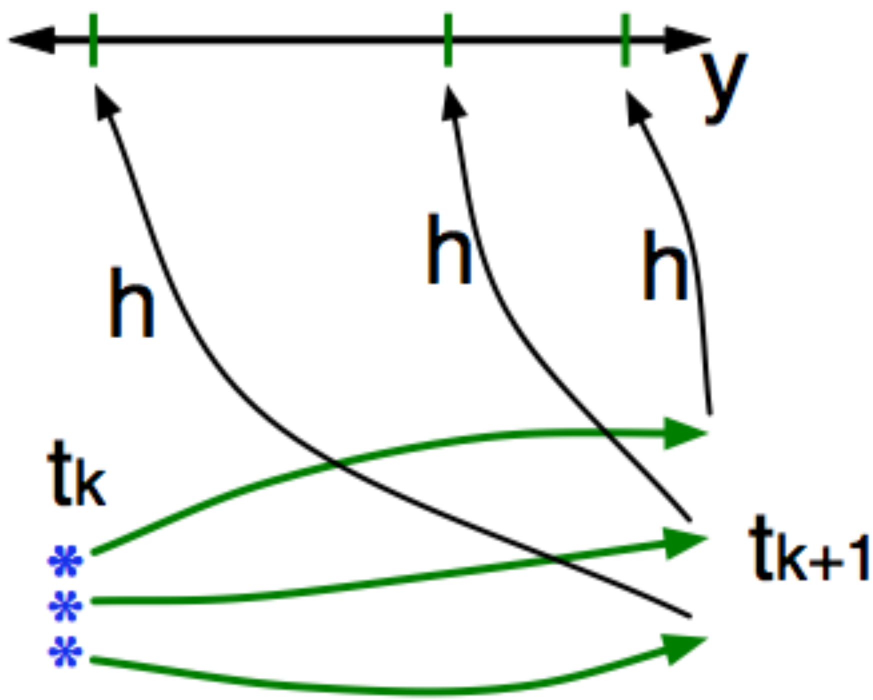
1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.

Ensemble state estimate after using previous observation (analysis)

Ensemble state at time of next observation (prior)

Ensemble Filter for Large Geophysical Models

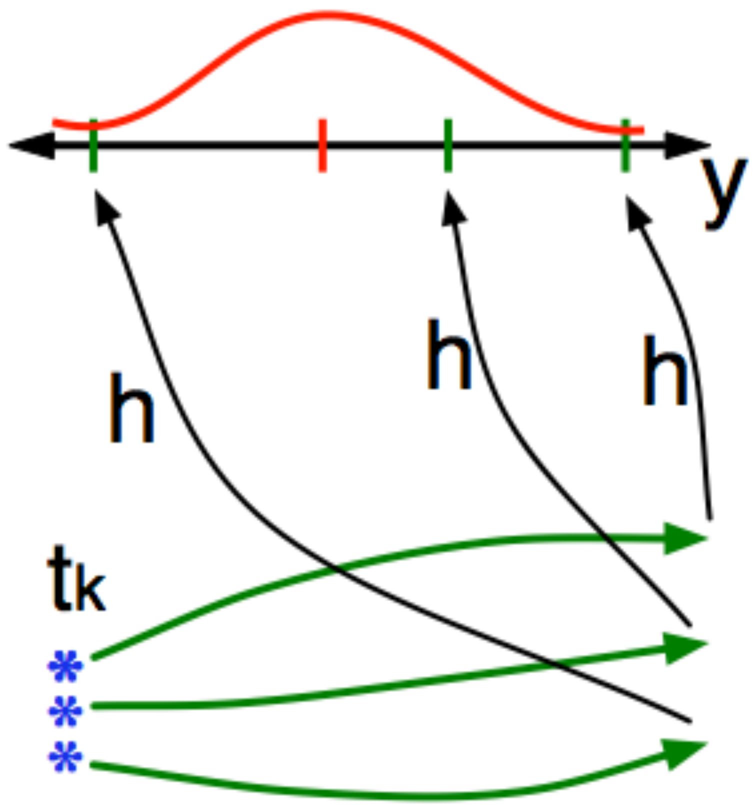
2. Get prior ensemble sample of observation, y = h(x), by applying forward operator **h** to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially.

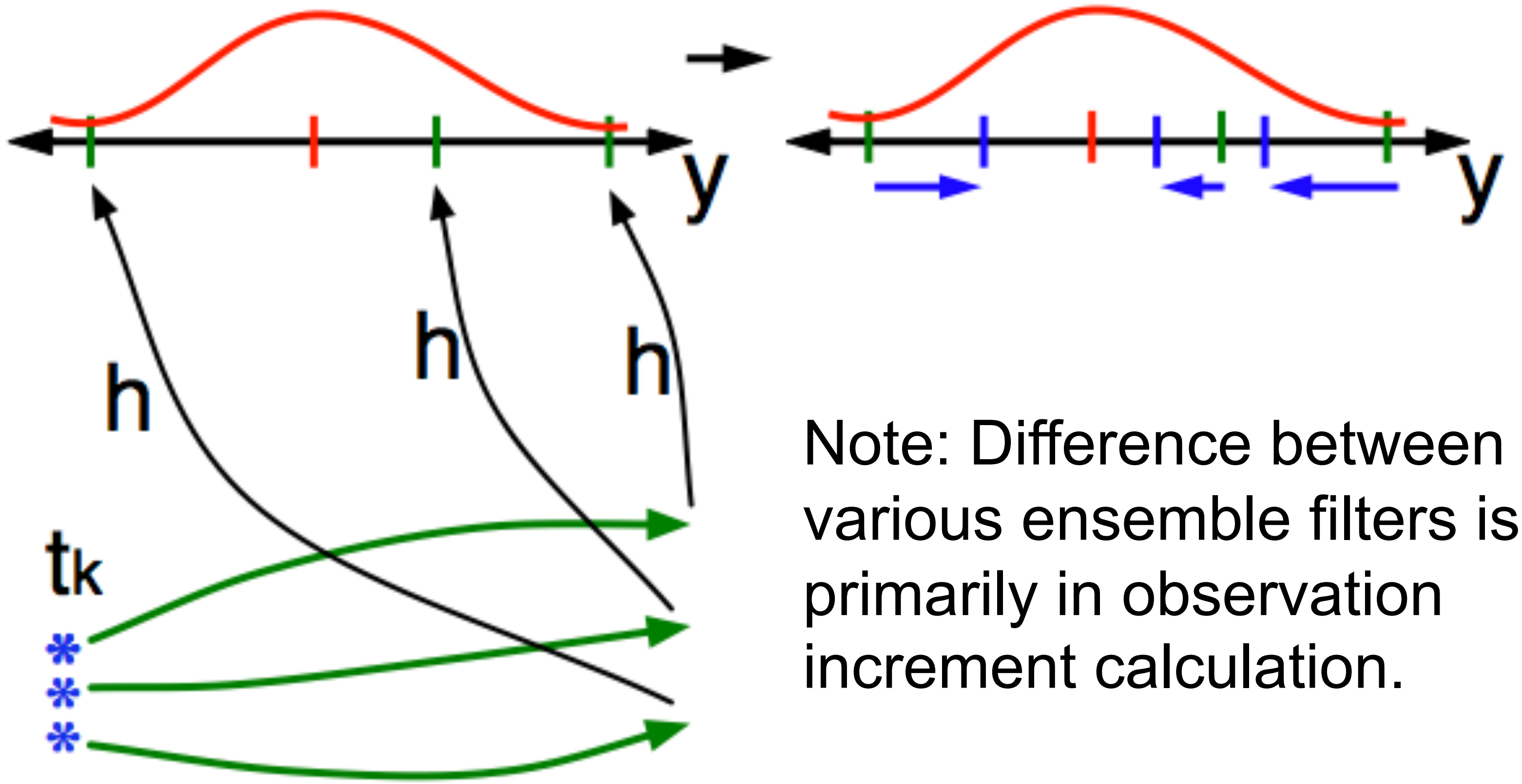
Ensemble Filter for Large Geophysical Models

3. Get observed value and observational error distribution from observing system.



Ensemble Filter for Large Geophysical Models

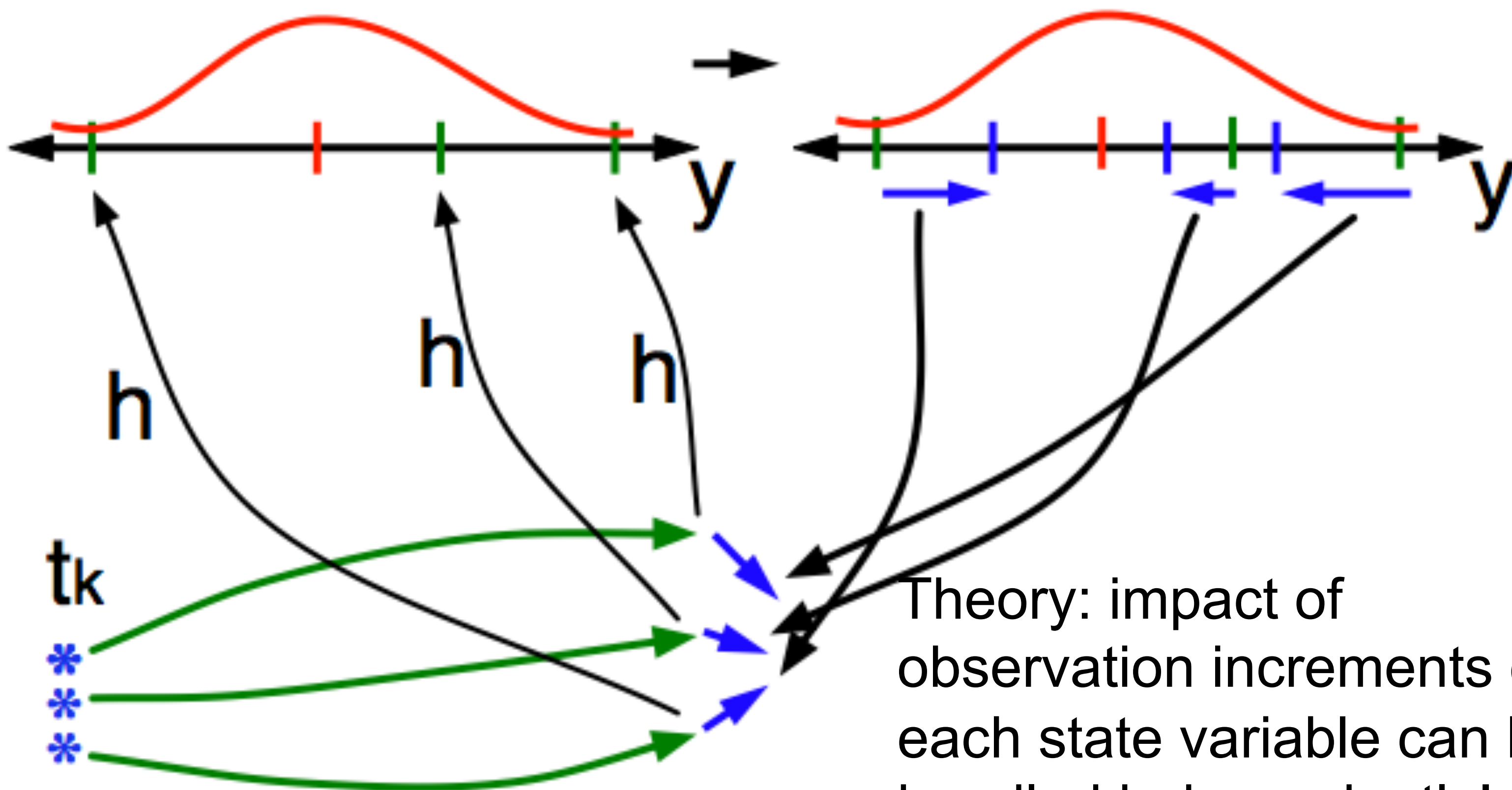
4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



Note: Difference between various ensemble filters is primarily in observation increment calculation.

Ensemble Filter for Large Geophysical Models

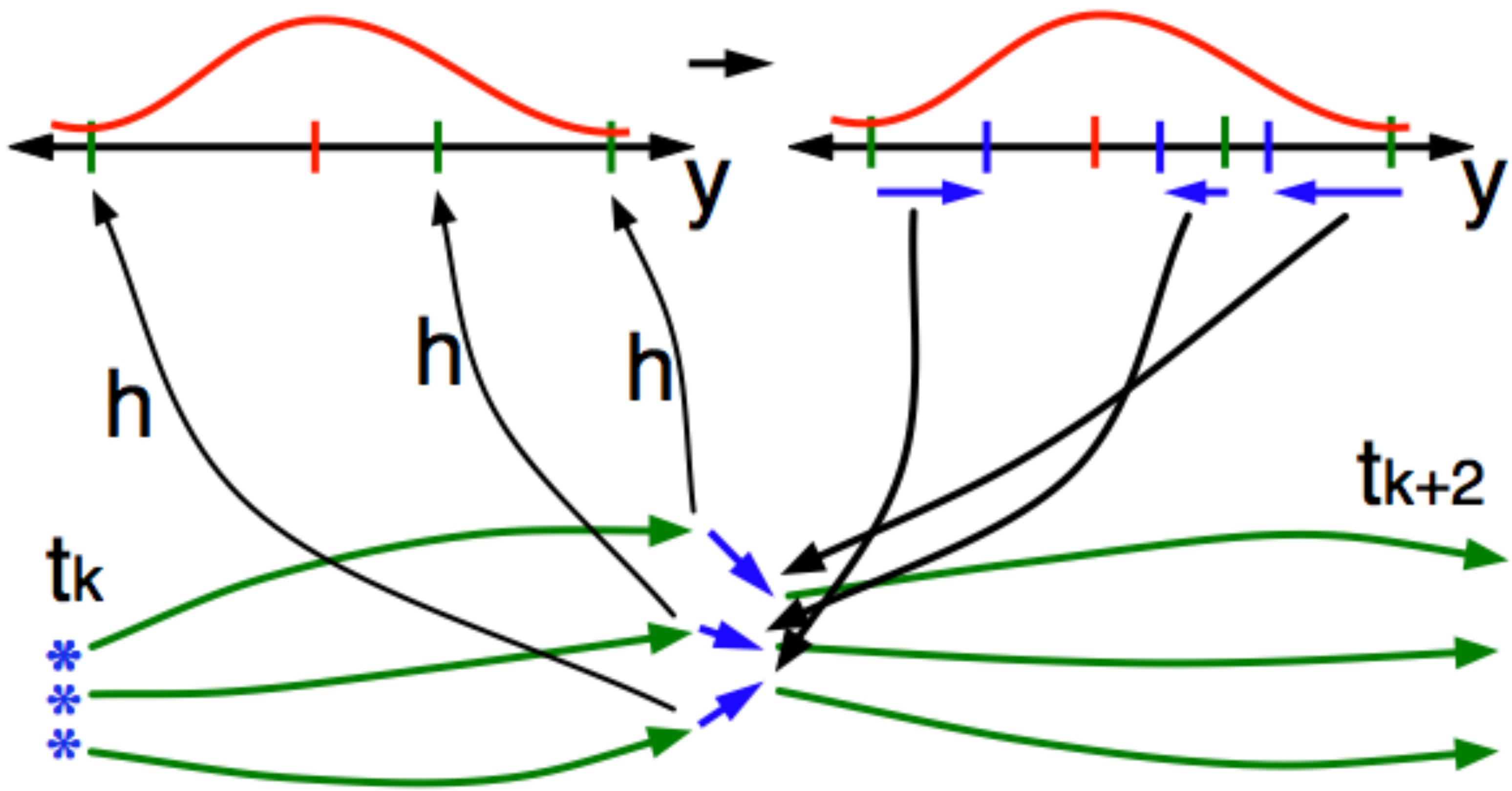
5. Use ensemble samples of **y** and each state variable to linearly regress observation increments onto state variable increments.



Theory: impact of observation increments on each state variable can be handled independently!

Ensemble Filter for Large Geophysical Models

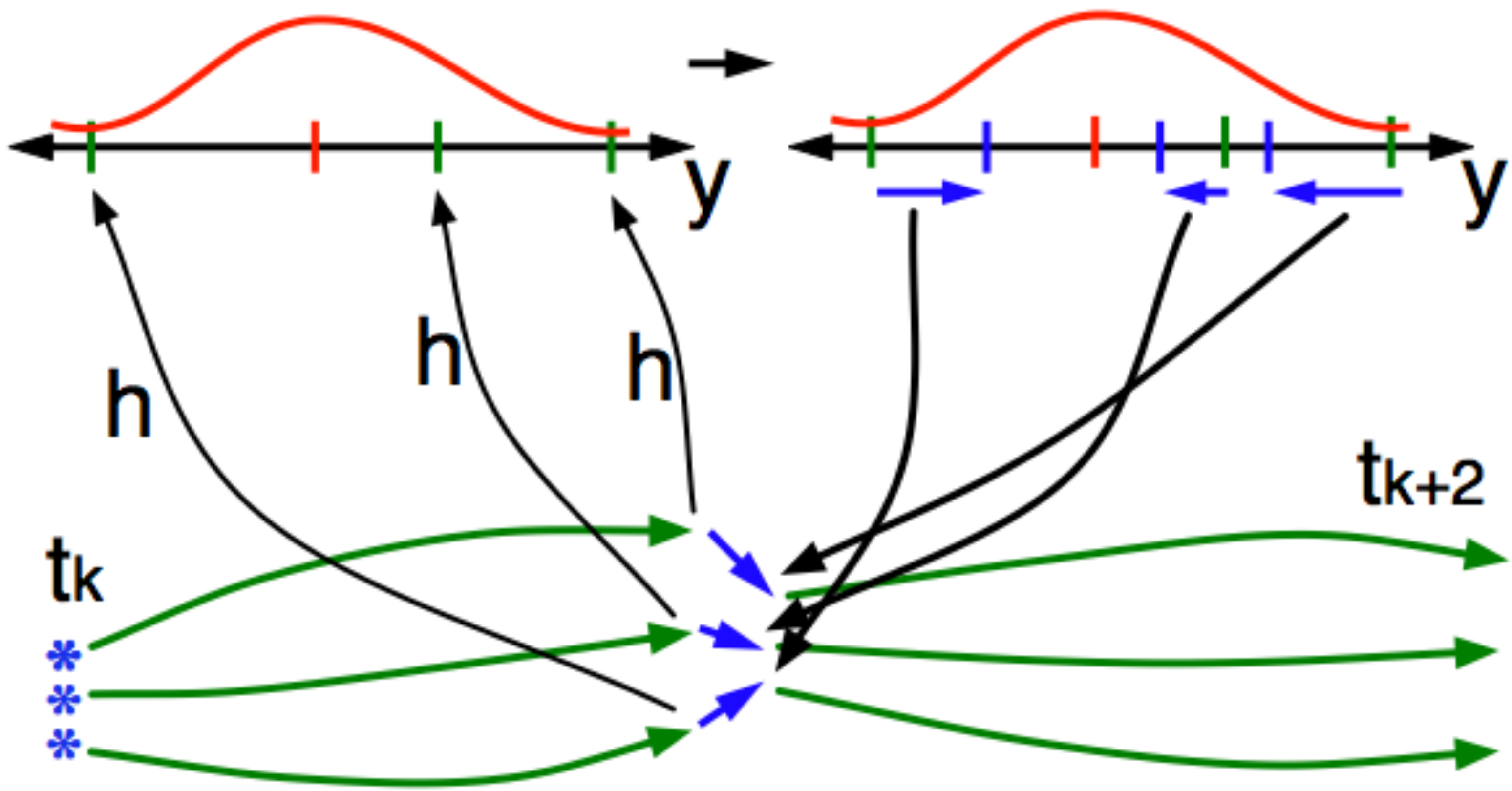
6. When all ensemble members for each state variable are updated, there is a new analysis. Integrate to time of next observation …



Ensemble Filter for Large Geophysical Models

A generic ensemble filter system like DART just needs:

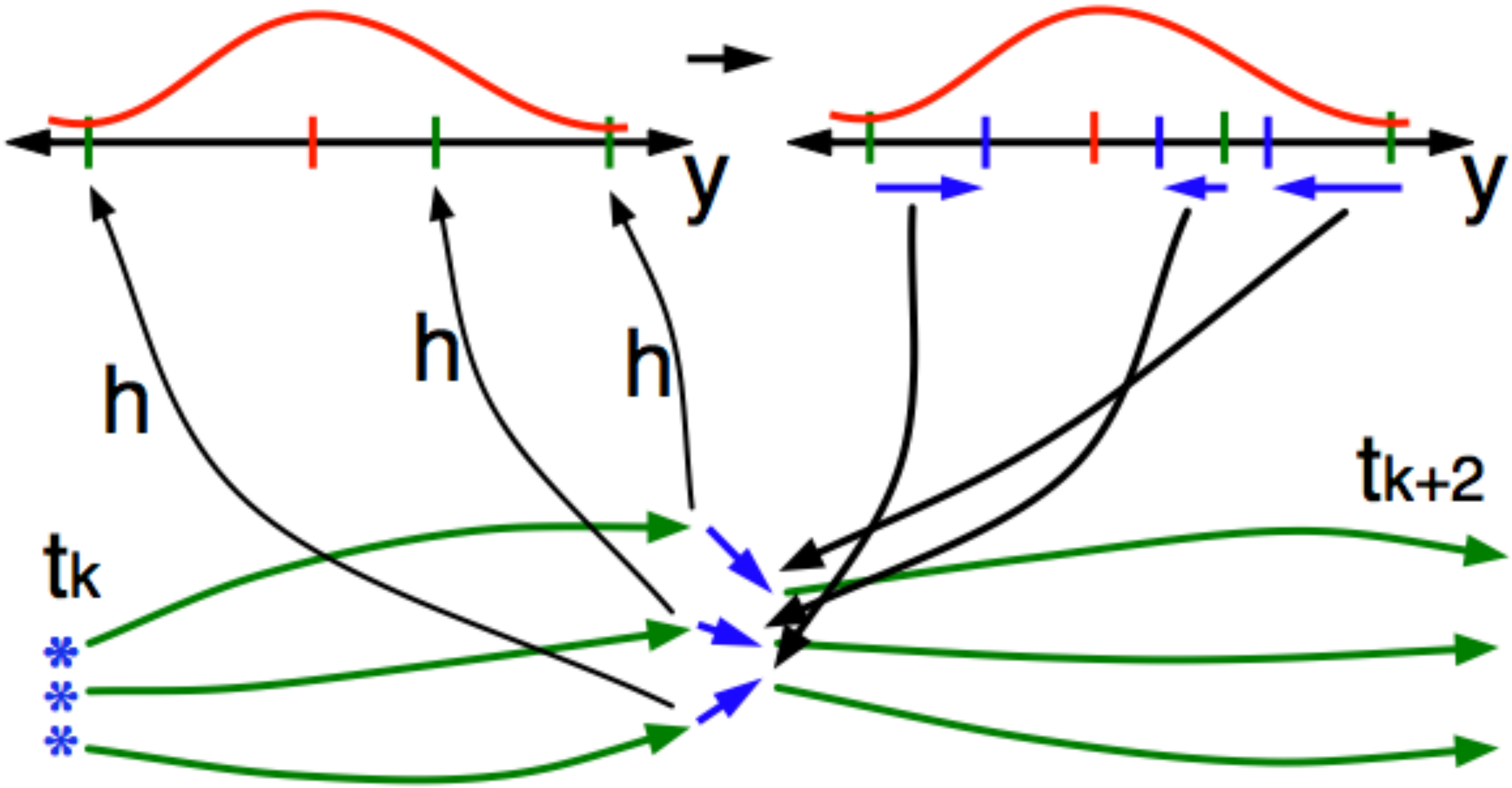
1. A way to make model forecasts;



Ensemble Filter for Large Geophysical Models

A generic ensemble filter system like DART just needs:

1. A way to make model forecasts;
2. A way to compute forward operators, h.



Ensemble Filters for Aerosols: Strengths

1. Fully multivariate:

All observations impact all state variables,
Tracer obs impact tracer and meteorological state,
Meteorological observations impact tracer state, too.

2. Tracers are modeled and assimilated ‘on-line’.

3. Complex forward operators (e.g. radiances) can be used.

Ensemble Filters for Aerosols: Challenges

1. Forecast model must generate covariances:
   Requires a good parameterized source model,
   Vertical distributions of tracer must be accurate.

2. Maintaining sufficient variability (spread).

3. Dealing with highly uncertain distributions.

4. Systematic errors in remote sensing observations.

Data Assimilation Research Testbed

DART is:

- Public domain software for Data Assimilation
  - Well-tested, portable, extensible, free!
- Models
  - Toy to HUGE
- Observations
  - Real, synthetic, novel
- An extensive Tutorial
  - With examples, exercises, explanations
- People: The DAReS Team

used at -

UNIVERSITY OF WASHINGTON

CIRES

THE UNIVERSITY OF UTAH

Central Weather Bureau

GFDL Geophysical Fluid Dynamics Laboratory Princeton, NJ

NOAA

PENNSTATE

California Institute of Technology

Jet Propulsion Laboratory
California Institute of Technology

and many more ...

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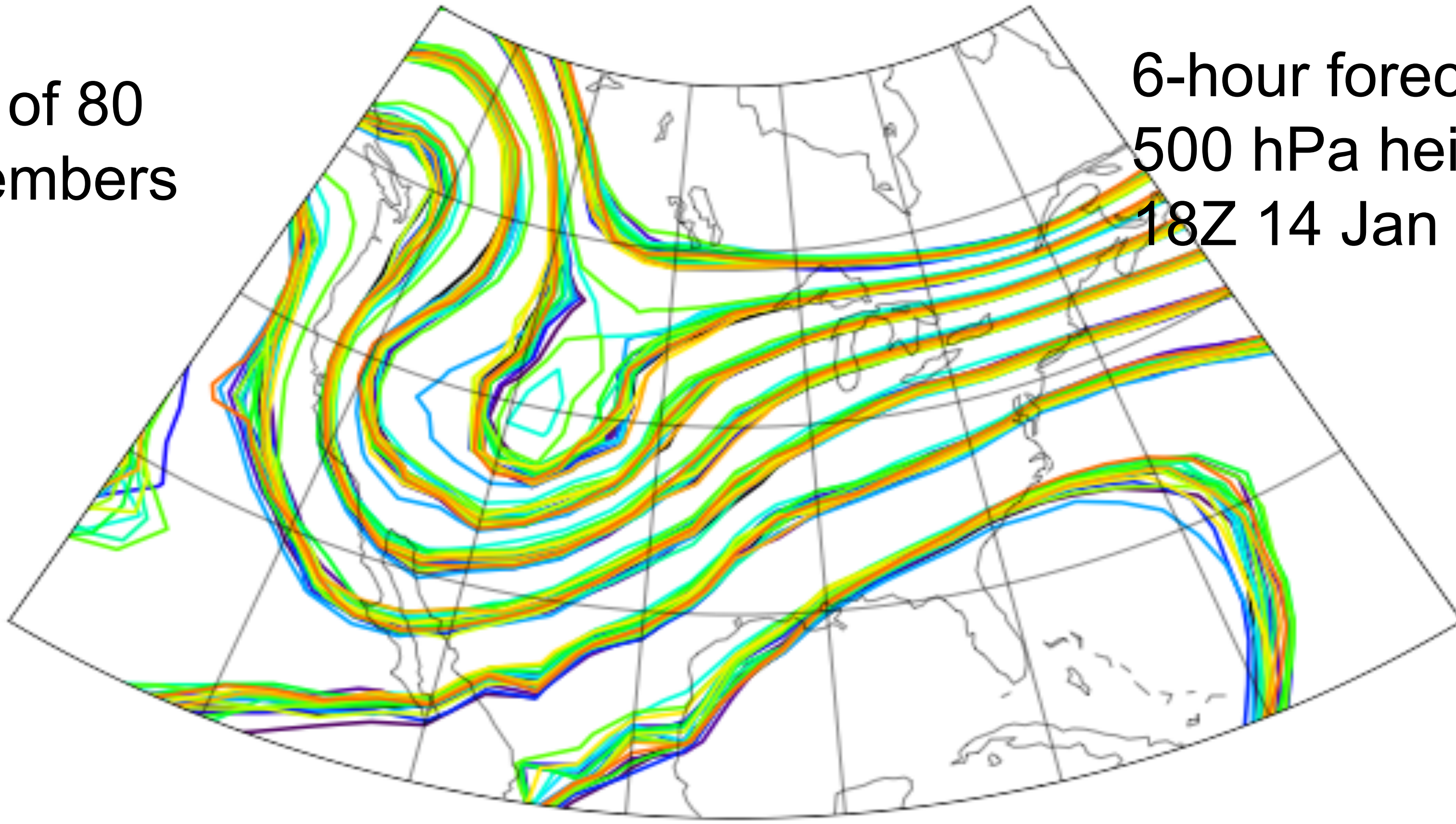
Basic Capability: Ensemble Analyses and Forecasts in Large Geophysical Models

20 of 80 members

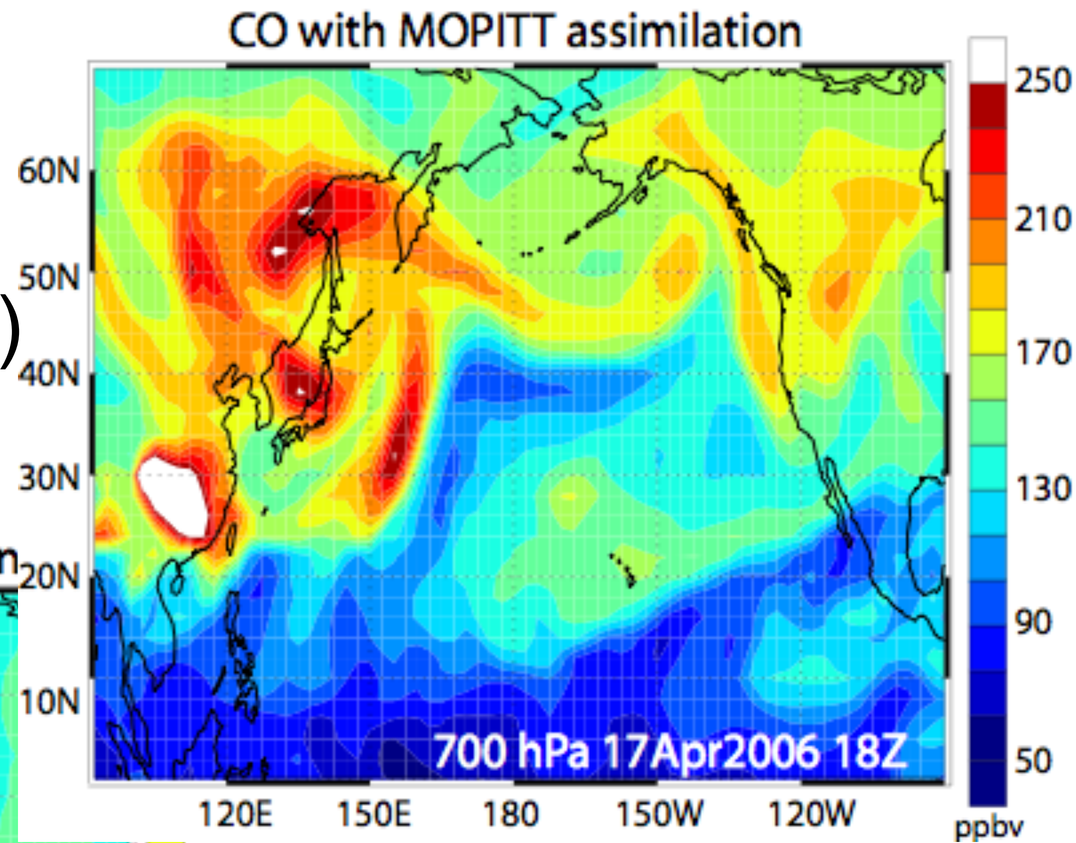
6-hour forecast
500 hPa height
18Z 14 Jan 2007

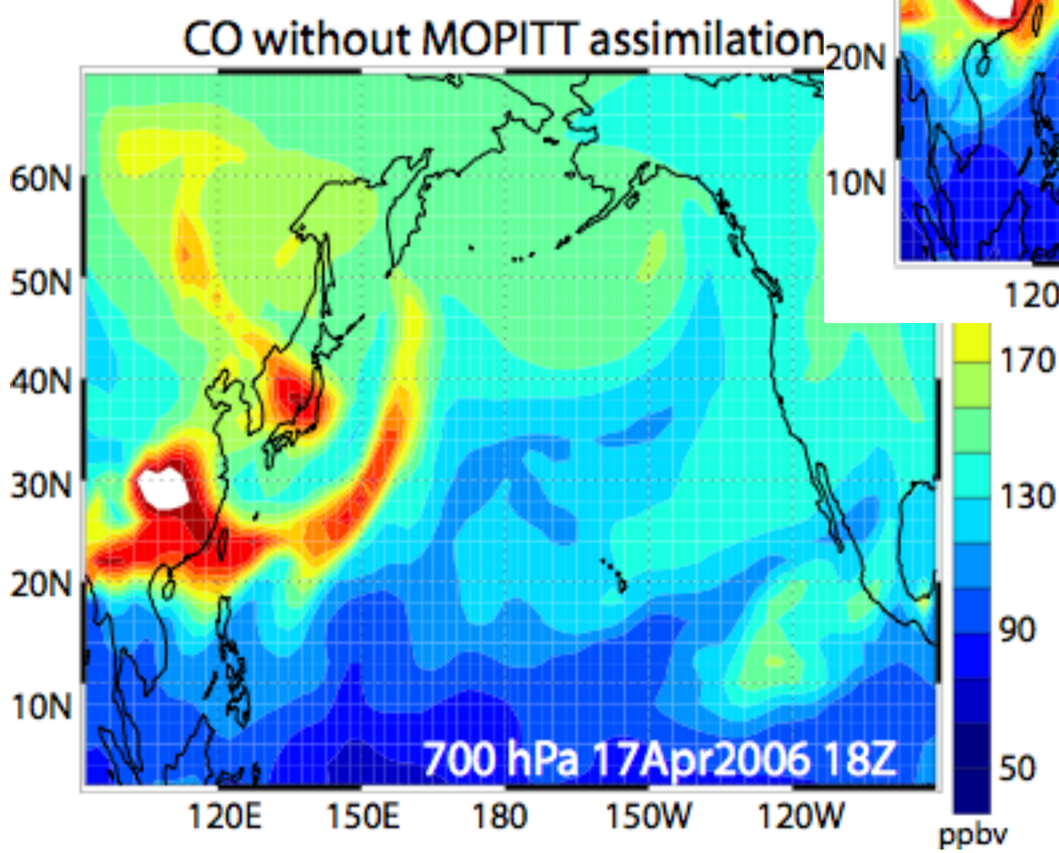


contours from 5400 to 5880 by 80

Forecast from CAM (Community Atmosphere Model)

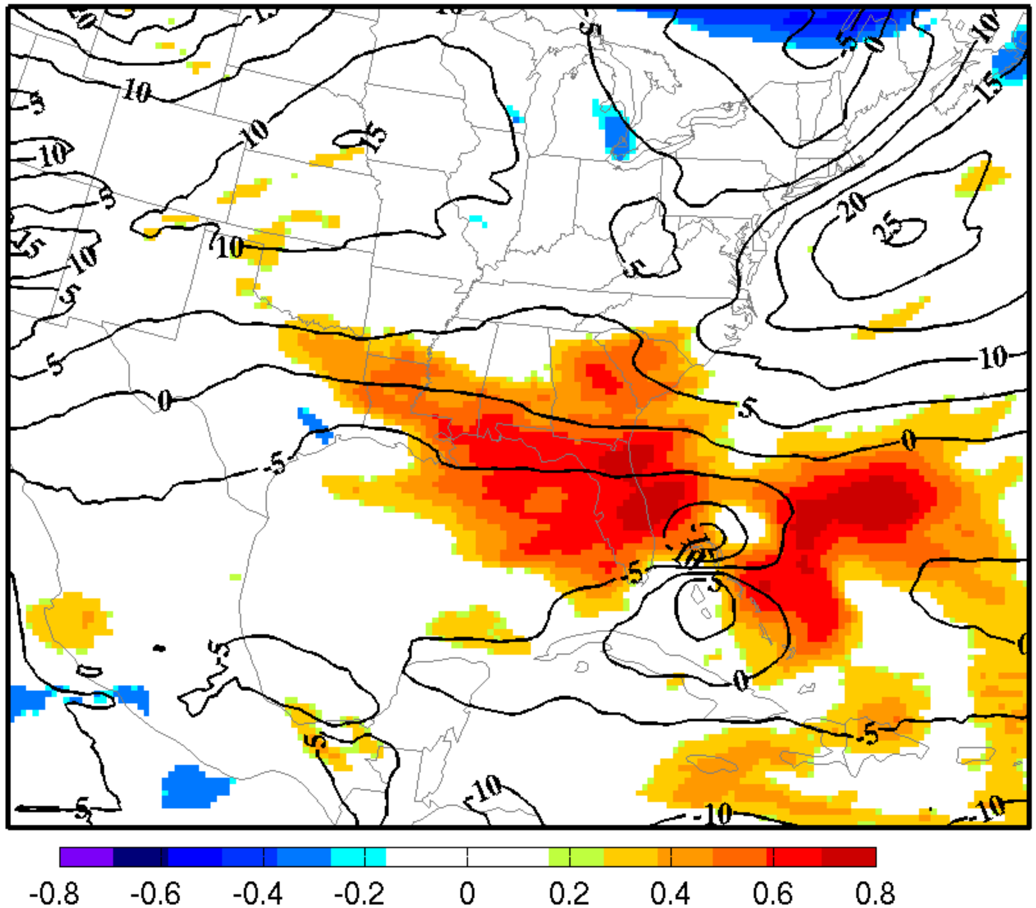
MOPITT CO assimilation prototype (CAM/CHEM model)





Support for ARCTAS field experiment

Hurricane Katrina Sensitivity Analysis



Contours: ensemble mean 48h forecast of deep-layer mean wind.

Color indicates change in the longitude of Katrina.

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Data Assimilation Research Testbed

It’s Easy to Add New Models (Tracers) to DART

Uses set of well-defined interfaces.
Adding a large global model takes person weeks.

Adding tracer to large model can be trivial.
Can add new tracer to CAM (global atmosphere model) at runtime.

Major DART compliant models

CAM spectral and FV, CAM/CHEM, WACCM

WRF, WRF/CHEM, WRF/MARS

GFDL AM2

CMAQ (EPA dispersion model)

POP ocean GCM

MIT ocean GCM

COAMPS

NOGAPS

ROSE (upper atmosphere model)

It's Easy to Add New Observations, too.

Requires only forward operator, h:
    maps state to expected observation.
No linear tangents or adjoints.
Limited amount of additional coding in well-defined framework.

DART Observation Types include:

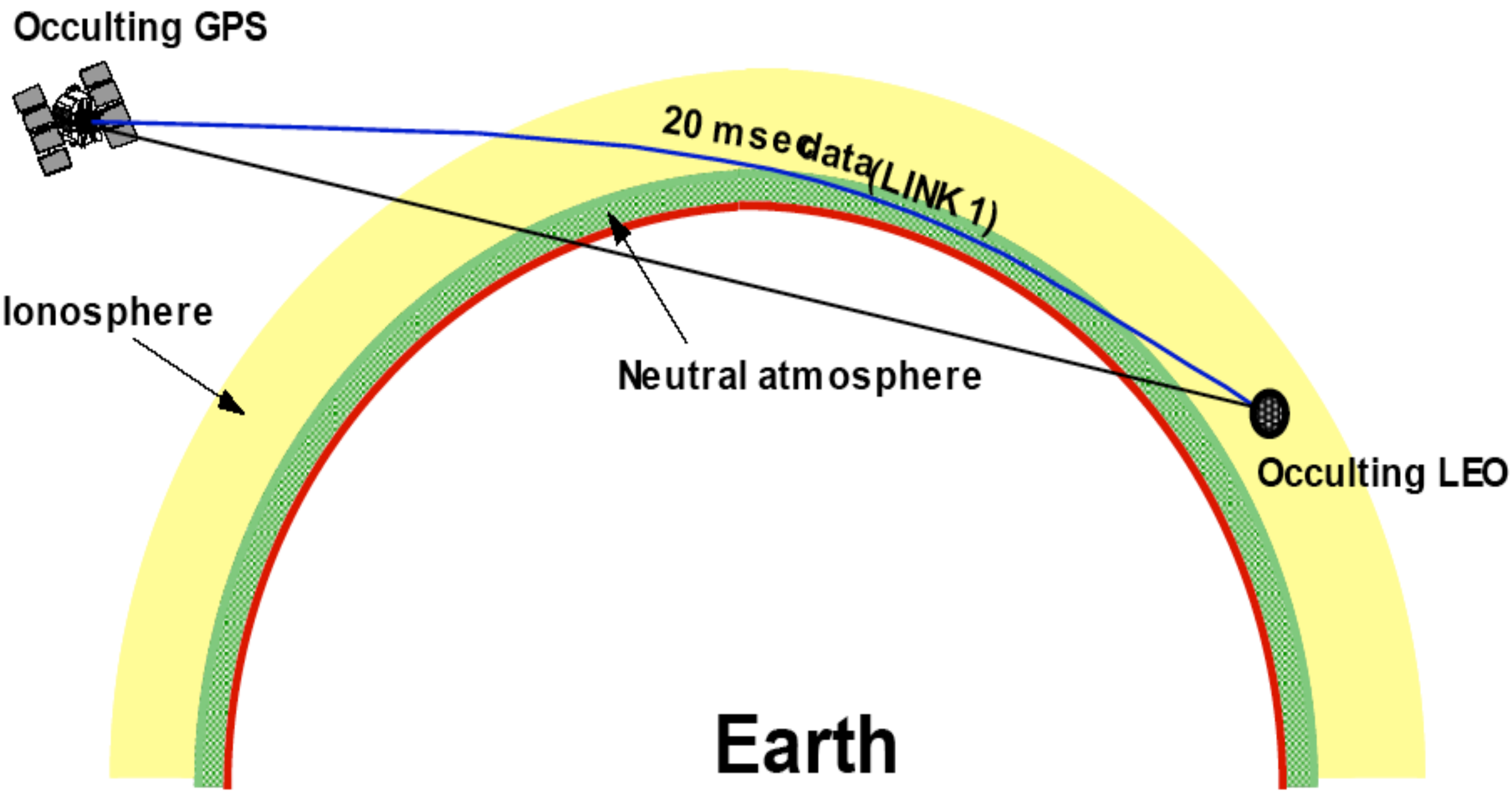
1. T, winds, moisture from radiosondes, ACARS.
2. Satellite drift winds.
3. Doppler radar velocity, reflectivity.
4. GPS radio occultation refractivity.
5. Ground-based GPS.
6. Scatterometer winds.
7. Retrievals from orbiting radiometers.
8. Development underway for radiances.

Aerosol observations easy to implement, challenging to use…

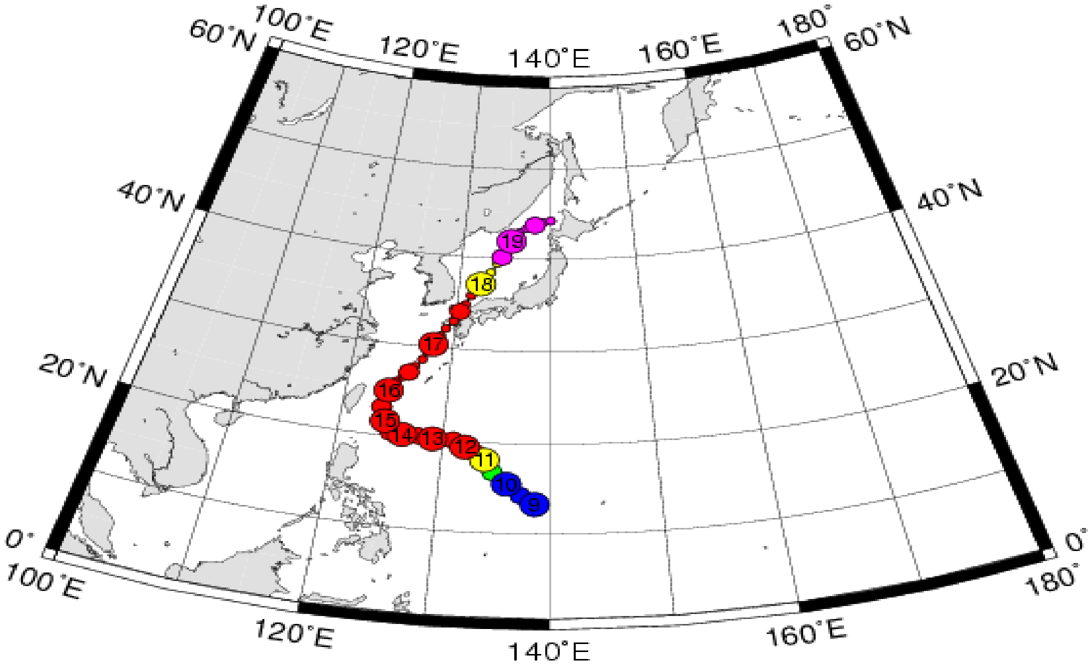
GPS Radio Occultation (RO)

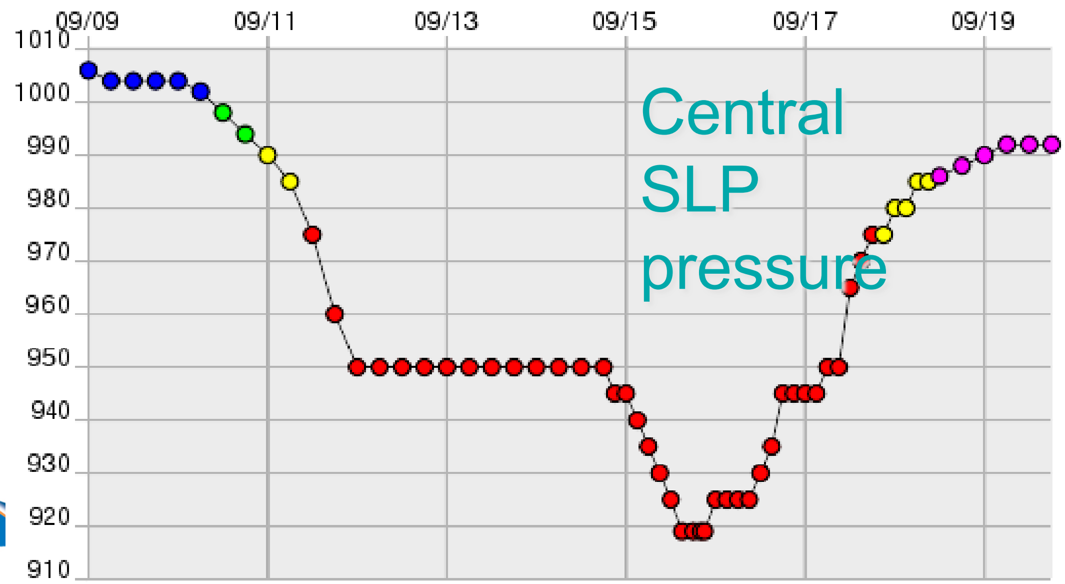
Basic measurement principle:

Deduce atmospheric water vapor and temperature based on measurement of GPS signal phase delay.



Typhoon Shanshan (Sept 10-17, 2006)



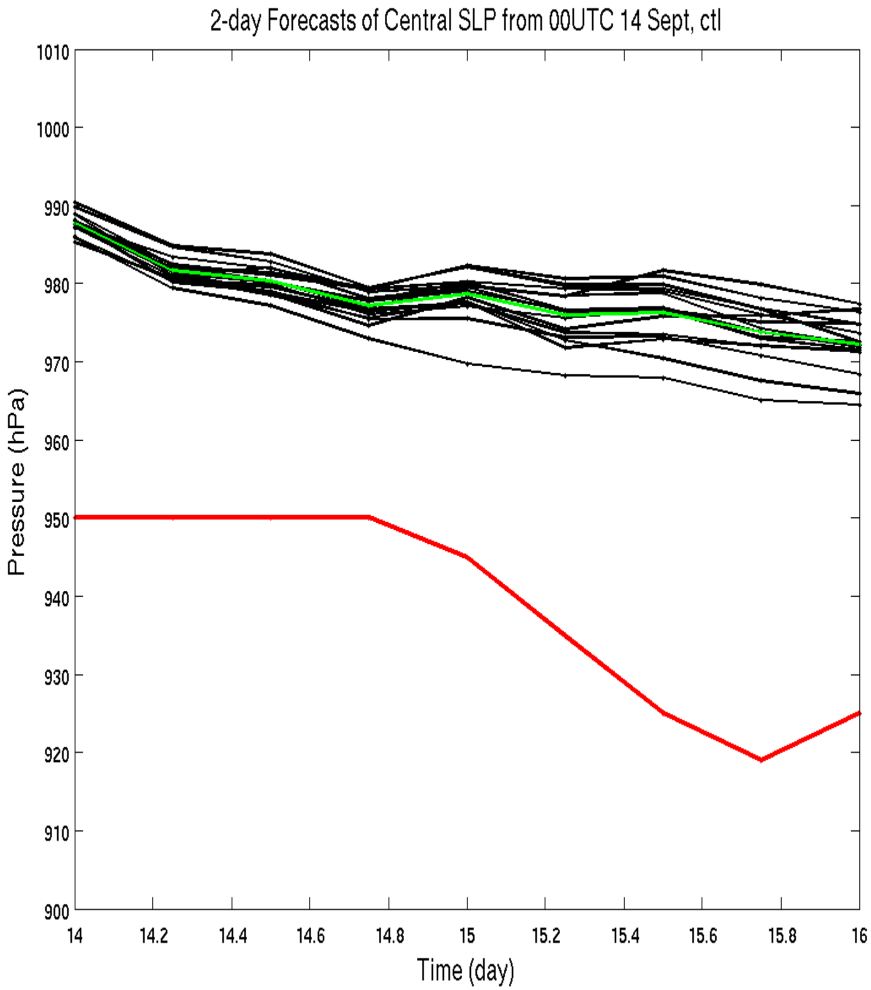


Operational forecasts using variational assimilation failed to predict the curving of the typhoon.

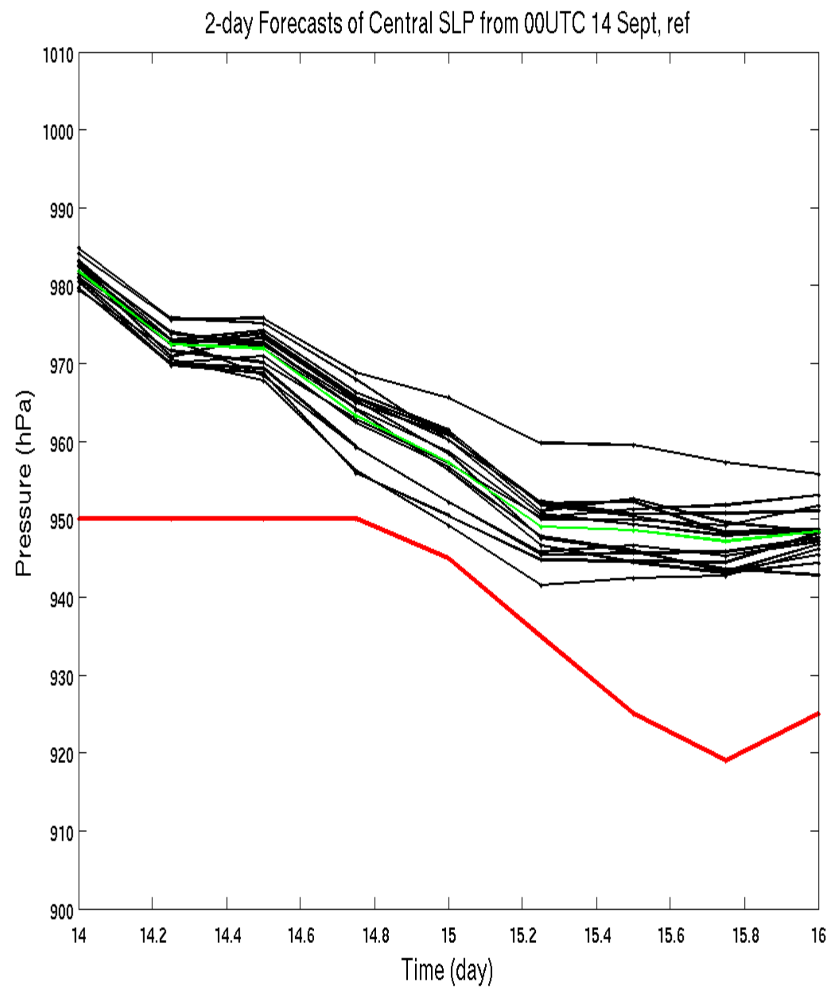
Data Assimilation Research Testbed

Ensemble Forecasts of Minimum Sea Level Pressure

NoGPS



GPS



Intensity of the typhoon is significantly increased with RO data.

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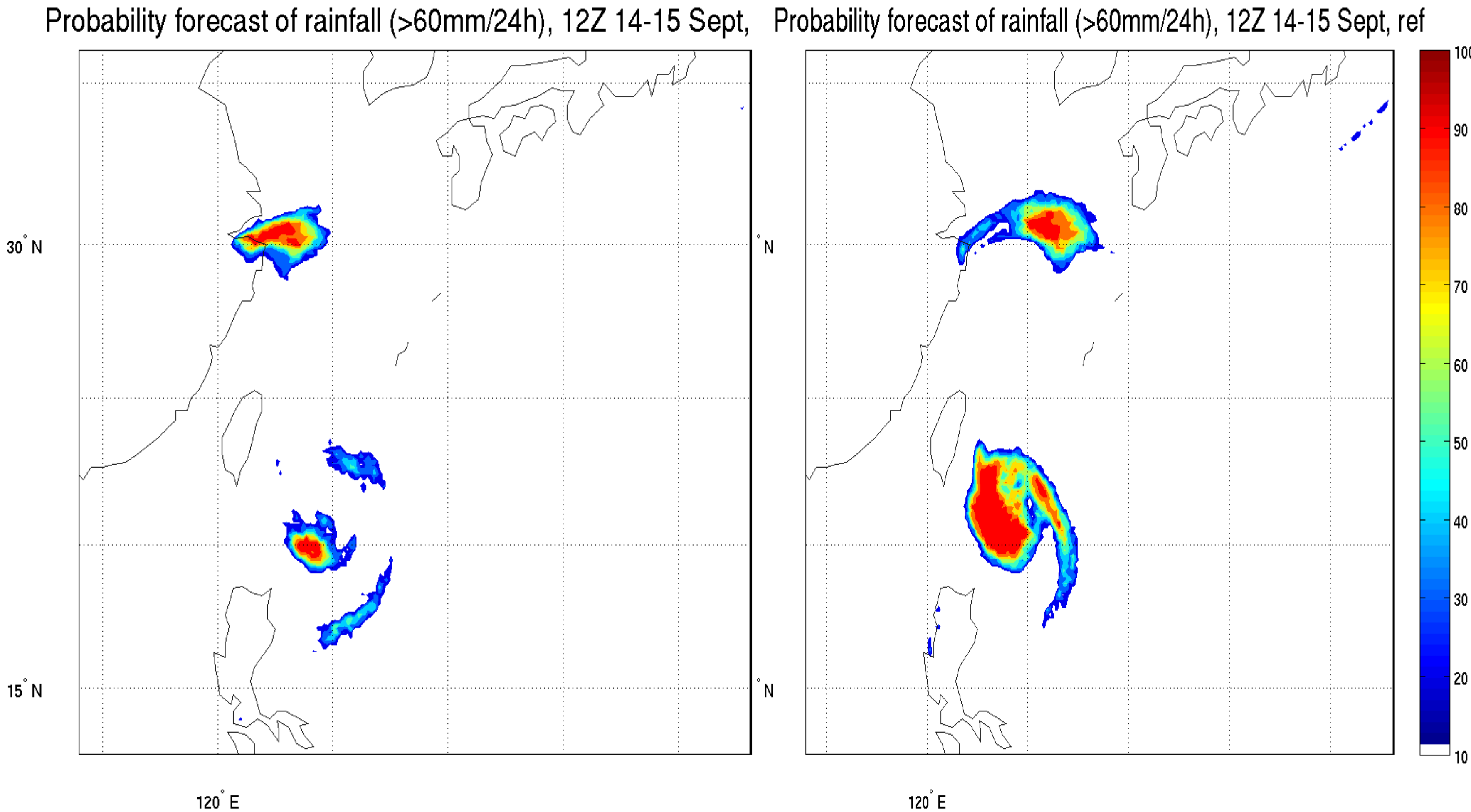
Data Assimilation Research Testbed

Forecast Probability of Rainfall >60mm/24h, 12Z 14-15 Sept.

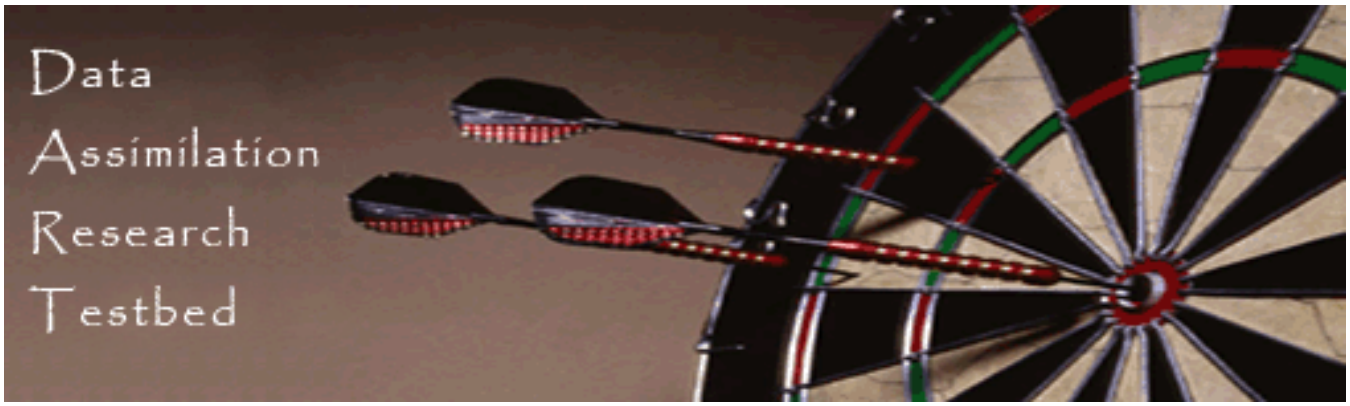
NoGPS GPS



Rainfall probability is increased with RO data.

Data Assimilation Research Testbed

Code to implement all of the algorithms discussed are freely available from:



http://www.image.ucar.edu/DAReS/DART/